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DISCUSSIONS

APPLICATIONS FOR ADMISSION AND TRANSFER

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DESIGN OF LARGE PIPE LINES

BY HERMAN SCHORER,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The object of this paper is to present an exact theory for the design of large pipe lines and their supports, which may also be used for the design of horizontal tanks, under-water tubes supported on piers, etc. Instead of the saddles commonly used, the pipe is supported and kept in shape by full ring girders which were first introduced in Sweden.

The ring girders act as stiff, disk-shaped members, preventing large distortions over the supports and their transmission into the pipe shell proper. Under these conditions stresses can be analyzed by the modern elastic theory of thin shells. It is found that the shell then is mainly subject to direct beam and ring stresses, the loads being transmitted to the supporting rings by shearing stresses. A narrow zone of secondary bending stresses occurs in the shell portions adjacent to the supporting rings, induced by the restraining action.

Theory and experience show that a pipe line so designed does not require any intermediate stiffening angles, even in case of comparatively thin shells. Much larger spans than those permitted by the customary saddle supports are entirely feasible, and considerable savings in steel and excavation may be realized with a larger factor of safety. The governing design stresses in the shell are derived and, for completeness, the stress analysis of a circular supporting ring is also given.

INTRODUCTION

Large steel pipe lines of usual design, supported on saddles and subject to beam action, are typified by short spans and a large number of circumferential stiffeners. Although the cost of such conduits is by no means small, present design practice is still largely based on crude, empirical rules, derived from observations on the unsuccessful behavior of some of the earlier structures, especially during filling and when operating under low pressure.

NOTE.—Written discussion on this paper will close in December, 1931 *Proceedings*.

[1] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

A radical departure from the old type saddle support, first introduced in Sweden about 1911, has been described by K. I. Karlsson[2] in an interesting paper. The pipe shell is riveted to and supported by circular girders, which transmit the load to the ground through a two-point support near the ends of the horizontal diameter. Tests and observations made on pipe lines thus supported have demonstrated the remarkable stability of the thin pipe shell built without any intermediate stiffening rings, and combined with unusual spans. For instance, the Kungfors line, built in 1921, has a shell thickness of $\frac{1}{4}$ to $\frac{5}{16}$ in., a diameter of almost 11 ft., with 41-ft. spans, and no stiffening angles between the ring-girder supports.

During the development stage of the Swedish type of support the elastic theory of thin shells had not advanced sufficiently to furnish a theoretical explanation for the small shell deformations under low heads. Although Karlsson has made an attempt to analyze the bending stresses in the thin shell, for the pipe full and half full, his assumptions were necessarily of a rather arbitrary nature, so that the uncertainty concerning the actual stress condition in the pipe shell was probably one of the reasons why the stiff-ring girder support has not replaced the saddle support in other countries.

Since the publication of Karlsson's paper, the general theory of shells has progressed to such an extent that the exact stress condition can be derived in a fairly simple manner. In the following analysis the pipe is considered as a thin shell, unsymmetrically loaded.

Notation

The following notation is used throughout this paper. Units must be consistent, for instance, inches, pounds, pounds per square inch, pounds per cubic inch, etc.

a = eccentricity of reaction, $\frac{Q}{2}$ (see Fig. 6), from a tangent to the center of the pipe shell.
b = moment arm of the tangential shear, s, from Point D in Fig. 6.
c = width of circular girder ring (see Fig. 4 (a)).
d = radial deflection:
- d_H = radial deflection accompanying the radial thrust, H, on the free cylinder rim.
- d_M = radial deflection caused by a radial moment, M, at a free rim.

e = 2.7183 = base of natural logarithms.
f = unit stress:
- f'_r = maximum ring stress in shell, pipe precisely full.
- f_r = maximum combined ring stress in shell, pipe under pressure.
- f_{bx} = unit longitudinal bending stress in the rim at a distance, x, from the rim (see Fig. 4(b)).
- f_L = combined maximum longitudinal beam stress.
- f_{bo} = maximum longitudinal rim bending stress in the shell.
- f_{bm} = absolute maximum value of the longitudinal rim bending stress.

[2] "Ueber Schwerkraftspannungen in Rohrleitungen von grossen Durchmessern und deren rationelle Konstruktion," by Karl I. Karlsson, *Schweizerische Bauzeitung*, Vol. 80, September 2, 1922, p. 105.

h = head.
l = elementary length; dl = segment lengths of a ring element (see Fig. 2).
m = Poisson's ratio.
p = variable pressure on the inside of the pipe circumference.
q = unit weight of the fluid flowing in a pipe.
r = mean radius of a pipe.
r'_p = increase in radius, r, at unrestricted sections of pipe.
r'_d = increase in radius, r, of the ring disk.
s = tangential shearing force per unit width of shell element.
t = thickness of shell.
u = angular location of a cylinder element, in the cross-section of a pipe, measured from the vertical (Fig. 2); du = angle subtended by the arc element, dl.
v = angular location of the point of tangency of s, measured from the vertical.
w = weight of pipe shell, per unit of area.
x = distance from shell rim (see Fig. 4(b)).
x_1 = abscissas in the cross-section of a pipe, to any point (D, Fig. 5) in the supporting ring.
x' = distance along the center line of the pipe to any point measured from the origin, O (see Fig. 1).
dx' = length of an arc element, dl wide, measured along the axis of the pipe.
y = ordinate of any point (D, Fig. 5) in a supporting ring.
z = a constant for the cylindrical shell = $\dfrac{\sqrt{r\,t}}{\sqrt[4]{3\,(1-m^2)}}$.
A_r = area of supporting ring in a plane along the axis of the pipe.
D = diameter of pipe = $2\,r$ (see Fig. 3).
E = modulus of elasticity.
H = uniformly distributed radial thrust (see Fig. 4), measured per unit width of rim.
L = length of span, from center to center of ring-girder supports.
M = bending moments in ring girder:
M_e = moment of external forces.
M_s = moment due to tangential shearing forces.
M_A = moment due to left reaction (Fig. 6); M_B = moment due to right reaction.
M_0 = moment at the top of the ring when u = zero.
M_a = ring moment at $u = \dfrac{\pi}{2}$.
M_1 = ring moment in the first quadrant; M_2, second quadrant; etc.
M_i = maximum intermediate moment.
M_x = moment at the elastic center of the supporting ring.
N = normal forces in ring girder:
N_0 = radial force at top of ring, when u = zero.
N_a = maximum ring force due to the shearing forces, s.
N_t = total tension in the ring caused by the combined loads, p and H.
P = external forces acting on a pipe.

Q = total load of the pipe shell, transmitted by shear to one supporting ring.
R = radius of curvature of a cylinder element (see Fig. 2).
S = section modulus of pipe, acting as a beam.
S' = section modulus of shell element.
T = stress components on shell element, measured per unit width:
T_1 = acting in a longitudinal direction.
T_2 = acting in a tangential direction.
X = longitudinal load component on shell element.
X_1 = horizontal force applied at the elastic center of supporting ring.
Y = tangential load component on shell element.
Y_1 = vertical force applied at the elastic center of supporting ring.
Z = radial load component on shell element.
δ = angular change:
δ_H = change that accompanies the deflection, d_H.
δ_M = change that accompanies the deflection, d_M.

Direct Stresses in Pipe Shell

Fig. 1 shows a horizontal pipe section connected to, and supported by, two stiff, disk-shaped members, which prevent large deformations of the rim and their transmission into the shell proper. Under such rim conditions the theory of thin shells shows that continuous surface loads, applied over the shell area, will cause mainly direct stresses in the pipe walls. The cylindrical shell acts

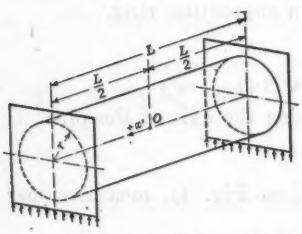

Fig. 1.

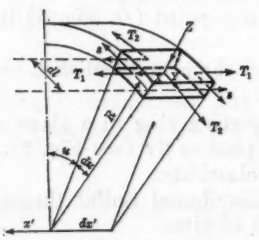

Fig. 2.

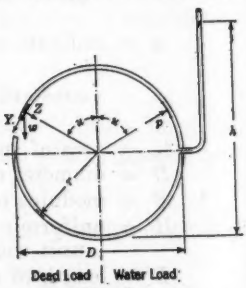

Fig. 3.

as a membrane, free from bending stresses normal to its surface, except for a narrow zone of secondary transverse stresses at the restrained rims. This behavior is quite in accordance with Castigliano's theorem of least work. Since a direct stress system is possible, much less internal work is required than with a system of large bending stresses in the pipe walls.

The following derivation of the direct stresses is based on the general theory of cylindrical shells, developed by Bauersfeld, and, later, independently by Finsterwalder, for the purpose of analyzing the stresses in the Zeiss-Dywidag, barrel-type roof, as described by Dischinger.[3]

Fig. 2 shows a small element of the pipe shell, loaded by the external force components, X, Y, and Z. The element is held in equilibrium by the stress components, T_1, T_2, and the shearing forces, s, which act in pairs of equal magnitude on all four faces. Designating tension by the positive

[3] Handbuch für Eisenbetonbau, Vol. 6, Fourth Edition: "Dachbauten Kragdacher, Schalen, und Rippenkuppeln," by H. J. Kraus and Franz Dischinger, p. 269, Wilhelm Ernst & Sohn, Berlin, 1928.

sign, and the radius of curvature of the cylinder element by R, Bauersfeld (as mentioned by Dischinger[3]) has derived the following general differential equations,

$$T_2 = - R Z \quad \ldots\ldots (1)$$

$$s = - \int \frac{\partial T_2}{R \partial u} dx' - \int Y dx' + f_1 (u) \quad \ldots\ldots (2)$$

and,

$$T_1 = - \int \frac{\partial s}{R \partial u} dx' - \int X dx' + f_2 (u) \quad \ldots\ldots (3)$$

The functions, $f_1 (u)$ and $f_2 (u)$, depend on the origin of the co-ordinate system and the conditions at the supports. With the origin at the center of the span, as shown in Fig. 1, $f_1 (u) = 0$. Assuming that any inside pressure on the end disks is resisted by corresponding forces in the opposite direction, then, in the case of Fig. 1, the stress component, T_1, must become zero at the supports, which condition determines $f_2 (u)$.

Equations (1), (2), and (3), can be integrated if the radius of curvature and the load components can be expressed as trigonometric functions of the angle, u. For a circular pipe shell, $R = r$ is constant.

The most important loadings are: First, the dead load of the shell; and, second, the pipe filled with a liquid, so that the free level just reaches the top of the shell.

Case 1.—Dead Load Stresses.—Designating the weight of the pipe shell per unit area by w, Fig. 3 gives the following values for the load components: $Z = w \cos u$; $Y = w \sin u$; and $X = 0$. Substituting Z in Equation (1), the ring stress component becomes,

$$T_2 = - r w \cos u \quad \ldots\ldots (4)$$

Substituting the partial derivative, $\frac{\partial T_2}{\partial u} = r w \sin u$, and the value of Y in Equation (2):

$$s = - 2 w x' \sin u \quad \ldots\ldots (5)$$

The partial derivative of s, $\frac{\partial s}{\partial u} = - 2 w x' \cos u$, is then substituted in Equation (3). By integration, $T_1 = \frac{w (x')^2}{r} \cos u + f_2 (u)$. Since $T_1 = 0$, when $x' = \frac{L}{2}$, $f_2 (u) = - \frac{w L^2}{4 r} \cos u$. Therefore,

$$T_1 = \frac{w}{r} \left[(x')^2 - \frac{L^2}{4} \right] \cos u \quad \ldots\ldots (6)$$

The unit stresses are obtained by dividing the expressions for the stress components by the shell thickness, t.

Equation (4) shows that the ring stress is independent of the span. The maximum values occur at the top (compression) and at the bottom (tension). The stress at the horizontal diameter is zero.

The shearing stresses, Equation (5), reach their maximum values at the end disks and are distributed over the cross-section as in a hollow, cylindrical beam, subject to bending. Equation (6) shows that the longitudinal shell stresses are also identical with those obtained by the ordinary theory of flexure, because cos u defines a straight-line stress distribution. Considering the shell as a simply supported beam, then, with the load assumptions previously given, the maximum moment is, $M_{max.} = \frac{2 \pi r w L^2}{8}$.

The section modulus of a thin circular ring is, $S = \pi t r^2$, therefore, the maximum beam stress,

$$f_{max.} = \frac{M_{max.}}{S} = \frac{w L^2}{4 r t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

By comparing Equation (7) with Equation (6), it may be noted that the maximum stresses are identical.

The direct longitudinal stresses, therefore, are the same as those in a hollow, cylindrical beam, but the addition of the ring stress, T_2, changes the principal stress system.

Case 2.—Stresses for Pipe Precisely Full.—Assuming that a liquid of the unit weight, q, precisely fills the pipe to the top, with the free level at $u = 0$; the load components are given by Fig. 3, as, $Z = -q r (l - \cos u)$; $Y = 0$; and $X = 0$.

From Equation (1),

$$T_2 = q r^2 (l - \cos u) \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Proceeding as shown for Case 1, from Equation (2),

$$s = -q r x' \sin u \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Equation (3) becomes,

$$T_1 = -\frac{q}{8}\left[L^2 - 4 (x')^2\right] \cos u \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Equation (8) shows that the ring stress is tension, and again is independent of the span. It increases from zero at the top to a maximum value at the bottom.

The shearing stress and the longitudinal stress are also identical with those obtained by the ordinary beam theory. It is interesting to note that the longitudinal stress due to liquid load is independent of the diameter.

Case 3.—Pipe Half Full.—A detailed analysis of the direct and secondary bending stresses in the pipe shell for this case of loading is given in a paper by A. F. Samsioe,[4] which came to the writer's attention after the present paper was submitted. The example given by Samsioe indicates that the maximum combined shell stresses for a pipe half full are of the same order

[4] "Die Spannungen in Einem auf Mehreren Stützen in Gleicher Gegenseitiger Entfernung Aufgelegten und zur Hälfte mit Wasser Gefüllten Rohr," by A. Frey Samsioe, pub. by Svenska Bokhandelscentralen, Stockholm, 1926.

as those obtained by the present method, when the direct stresses of Case 2 are combined with the rim bending stresses, as shown subsequently. For practical purposes, a design based on Case 2, including the rim stresses, will, therefore, also be safe for a pipe half full.

Case 4.—Combined Direct Stresses.—Combining the two cases of loading and designating the maximum unit ring stress by f'_r, then from Equations (4) and (8), with $D = 2\,r$,

$$f'_r = \frac{D}{2\,t}\,(w + q\,D) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

This stress occurs at the bottom of the shell, where the tension due to the dead load is increased because of the weight of the liquid.

If the pipe is under pressure, corresponding to a head, h, above the bottom of the pipe, then the maximum combined ring stress, designated by f_r, becomes,

$$f_r = \frac{D}{2\,t}\,(w + q\,h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The combined maximum longitudinal stress (f_L) is obtained from Equations (6) and (10); with $x' = 0$, and $u = 0$, or $u = \pi$,

$$f_L = \mp\,\frac{L^2}{4\,t}\left(\frac{2\,w}{D} + \frac{q}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It can be shown that, in case of continuous pipe lines, the direct longitudinal stresses can also be derived from the theory of continuous beams.

If the pipe is prevented from expansion and contraction due to temperature changes, the longitudinal stresses due to this cause must be superimposed to the stresses, f_L. However, by providing expansion joints and bearings of low frictional resistance the temperature stresses can practically be eliminated.

Ring buckling of the pipe shell must be investigated only in case of possible outside pressure, for instance, that due to partial vacuum. It will generally be more economical to avoid vacuum pressure by the installation of stand-pipes and relief valves. Buckling due to longitudinal compressive stresses in thin, hollow cylinders has been analyzed by Timoshenko.[5] It may be expected that no reduction of the usual design stresses will be required until the ratio, $\frac{r}{t} > 500$, which is a remote possibility in practical cases.

Longitudinal and circumferential joints of riveted and welded construction must be designed to transmit the full shell stress.

Rim Stresses

Assume that the pipe section shown in Fig. 1 is under uniform inside pressure. Due to the elasticity of the shell the pipe diameter will increase by a small amount in direct proportion to the ring stress. The shell rims, on account of their rigid connection with the end disks, cannot widen in the same manner as the more distant shell portions, so that bending deformations occur in the restrained rim zone.

[5] "Einige Stabilitätsprobleme der Elastizitätstheorie," by Timoshenko, *Zeitschrift für Mathematik und Physik*, 1910, p. 337.

In practical cases the pipe is built as a continuous structure over more than two supports. In order to provide an unrestricted flow area the full disks of Fig. 1 are replaced by ring disks members. Fig. 4 (*a*) shows the state of deformation of a pipe shell, which is under a uniform inside pressure, p, and is rigidly connected to a circular ring of constant cross-sectional area, A_r. The radius of the unrestricted shell portions has increased by the elastic amount, r'_p, and the radius of the ring disk, by the amount, r'_d. Fig. 4 (*b*) shows one-half the pipe shell cut at its connection with the ring. The original

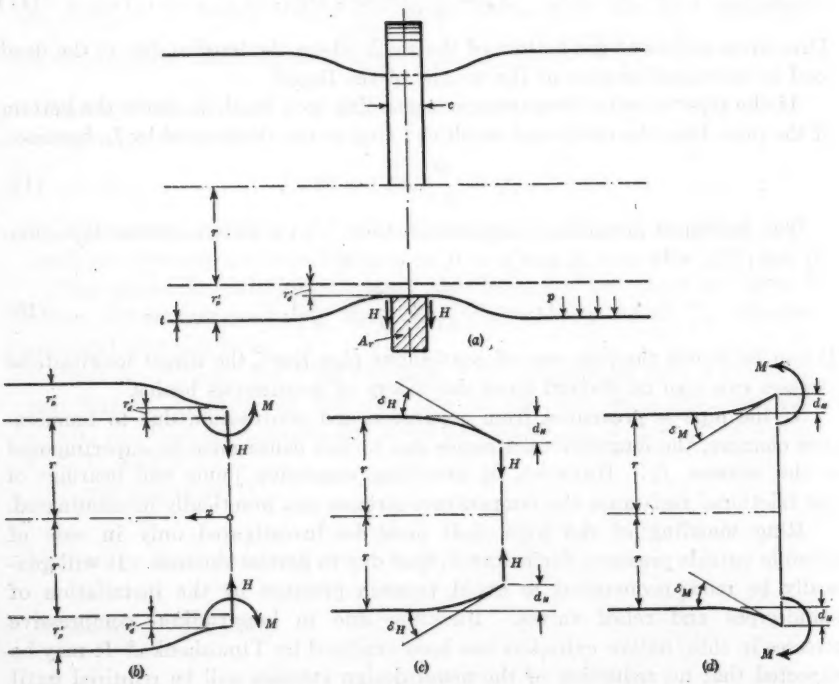

FIG. 4.

deformation of the free rim is restored by applying a uniformly distributed radial thrust, H, acting inward, and a uniformly distributed radial moment, M, which has the tendency to turn the rim outward. H and M are measured per unit length of circumference.

The rim deformations due to such radial loads are given by Meissner[6] and Pasternak.[7] The following analysis of the rim stresses is based on Pasternak's method.

If the free cylinder rim is loaded by the radial thrust, H, alone, as shown in Fig. 4 (*c*) the rim deformation is expressed by the radial deflection,

[6] "Zur Festigkeitsberechnung von Hochdruck-Kesseltrommeln," by E. Meissner, *Schweizerische Bauzeitung*, Vol. 86, July 4, 1925, p. 1.

[7] "Die praktische Berechnung biegefester Kugelschalen, kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischer Wandungen in gegenseitiger monolither Verbindung," by P. Pasternak, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, 1926, p. 1.

d_H, and the angular change of the generatrix, designated by δ_H. If the free rim is loaded by the radial moment, M, alone, as shown in Fig. 4 (*d*), the corresponding deformations are designated by d_M and δ_M, respectively.

Pasternak has derived the following expressions for the deformations of the rim, due to the radial loads, H and M:

$$d_H = \frac{2\,r^2\,H}{t\,z\,E} \qquad (14)$$

$$\delta_H = \frac{2\,r^2\,H}{t\,z^2\,E} \qquad (15)$$

$$d_M = \frac{2\,r^2\,M}{t\,z^2\,E} \qquad (16)$$

and,

$$\delta_M = \frac{4\,r^2\,M}{t\,z^3\,E} \qquad (17)$$

in which, z is a constant for the shell, equal to $\frac{\sqrt{r\,t}}{\sqrt[4]{3\,(1-m^2)}}$.

Due to the inside pressure, p, per unit area, the radius of the unrestricted shell portions increases by the amount,

$$r'_p = \frac{p\,r^2}{t\,E} \qquad (18)$$

The disk ring is loaded by the direct pressure, p, acting over the width, c, and the radial thrust, H, transmitted from both shell rims. Designating by N_t the total tension in the ring caused by the combined loads, then,

$$N_t = (p\,c + 2\,H)\,r \qquad (19)$$

Furthermore, if A_r designates the cross-sectional area of the ring,

$$r'_d = (p\,c + 2\,H)\,\frac{r^2}{A_r\,E} \qquad (20)$$

With reference to Fig. 4 (*b*), the following relation can next be written,

$$r'_p - d_H + d_M = r'_d \qquad (21)$$

Since the connection between the ring and the shell is assumed as fixed,

$$\delta_H = \delta_M \qquad (21a)$$

Substituting the corresponding values for the deformations, Equations (14), (15) (16), (17), (18), and (20), in Equations (21) and (21*a*), respectively,

$$H = p\,z\,\frac{(A_r - c\,t)}{(A_r + 2\,t\,z)} \qquad (22)$$

and,

$$M = \frac{p\,z^2}{2}\,\frac{(A_r - c\,t)}{(A_r + 2\,t\,z)} \qquad (23)$$

For steel, with $m^2 = 0.10$, the shell constant equals,

$$z = \frac{\sqrt{r\,t}}{\sqrt[4]{3\,(1-m^2)}} = 0.78\,\sqrt{r\,t} \qquad (24)$$

Therefore, Equation (22) becomes,

$$H = 0.78\, p\, \sqrt{r\, t}\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t\, \sqrt{r\, t})} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

and Equation (23),

$$M = 0.304\, p\, r\, t\, \frac{(A_r - c\, t)}{A_r + 1.56\, t\, \sqrt{r\, t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The section modulus, S', of the shell plate, per unit width of circumference, is $\frac{t^2}{6}$. Designating by f_{bo} the maximum rim bending stress of the shell, from Equation (26),

$$f_{bo} = \frac{M}{S'} = 1.82\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t\, \sqrt{r\, t})} \times \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots (27)$$

In Equation (27), the factor, $\frac{p\, r}{t}$, is identical with the wing stress in the unrestricted shell, also called the cylinder stress. The rim bending stress is, therefore, a function of the cylinder stress in the more distant shell portions. If A_r becomes very large, comparable to an absolutely unyielding disk, the rim bending stress approaches an absolute maximum value, designated by f_{bm}, for which Equation (27) gives,

$$f_{bm} = 1.82\, \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

In this extreme case the rim bending stress exceeds the cylinder stress by fully 82 per cent. The smaller the area of the ring disk, the smaller the rim bending stress will be. The bending stresses in the rim zone cannot be neglected in the design, because in continuously supported pipe lines the longitudinal stresses due to beam action are combined with the longitudinal rim bending stresses.

Equation (27) also shows the influence of the width, c, on the magnitude of the rim bending stress. The wider the ring disk, the smaller will be the bending stress for a constant area, A_r.

In all thin shells of revolution, subject to symmetrical rim loads, the bending stresses decrease rapidly with increasing distances from the rim in the shape of a damped wave, as shown by Geckeler.[8] Designating by f_{bx} the rim bending stress at the distance, x, from the rim,

$$f_{bx} = f_{bo}\, \sqrt{2}\, e^{\frac{-x}{z}} \cos\left(\frac{x}{z} + \frac{\pi}{4}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Equation (29) shows that the thinner the shell compared to its diameter, the narrower the rim zone of secondary bending stresses will be.

The total tension, N_t, in the supporting ring can now be determined by substituting H, from Equation (25), in Equation (19), which gives,

$$N_t = p\, r \left[c + 1.56\, \sqrt{r\, t}\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t\, \sqrt{r\, t})}\right] \quad \ldots\ldots\ldots\ldots (30)$$

[8] "Ueber die Festigkeit achsensymmetrischer Schalen," by J. Geckeler, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Heft 276, VDI-Verlag, Berlin, 1926.

This determination of the rim stresses applies only to symmetrical rim loads. Dischinger[9] has shown that in case of unsymmetrical loads the maximum rim bending stress is obtained with good approximation by substituting the maximum ring stress in place of the cylinder stress, $\frac{p\,r}{t}$, in Equation (27). In the present case, the maximum ring stress is given by Equation (12).

Stresses in Supporting Ring

Fig. 5 shows a circular ring of uniform cross-section, loaded and held in equilibrium by the external forces, P. For the purpose of analysis the ring is cut at the top and the right half is fixed at C'. The left half is provided with a rigidly connected lever arm, $C\,O$, in which, O represents the elastic center of the ring (identical in this case with the geometric center). At O, the unknown forces, X_1, Y_1, and the unknown moment, M_z, are applied so that the ring deformations at C, with respect to C', disappear.

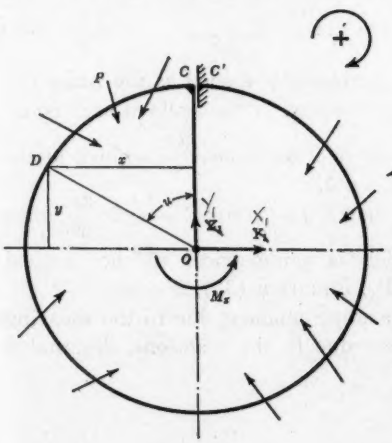

Fig. 5.

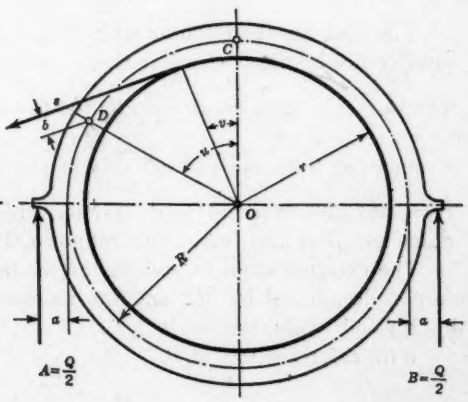

Fig. 6.

If dl designates the length of a small ring element and M_e defines the moment of the external forces between C and D with respect to D, then the unknowns are found from the following equations, as given by Mueller-Breslau:[10]

$$X_1 = \frac{\int M_e\, y\, dl}{\int y^2\, dl} \qquad (31)$$

$$Y_1 = \frac{\int M_e\, x\, dl}{\int x_1^2\, dl} \qquad (32)$$

and,

$$M_z = \frac{\int M_e\, dl}{\int dl} \qquad (33)$$

[9] Handbuch für Eisenbetonbau, Vol. 6. Fourth Edition, p. 247.

[10] "Die neueren Methoden der Festigkeitslehre," by H. Mueller-Breslau, pp. 125, 134, pub. by Alfred Kroener, Leipzig, 1913.

In practical cases the pipe is built as a continuous structure over more than two supports. In order to provide an unrestricted flow area the full disks of Fig. 1 are replaced by ring disks members. Fig. 4 (*a*) shows the state of deformation of a pipe shell, which is under a uniform inside pressure, p, and is rigidly connected to a circular ring of constant cross-sectional area, A_r. The radius of the unrestricted shell portions has increased by the elastic amount, r'_p, and the radius of the ring disk, by the amount, r'_d. Fig. 4 (*b*) shows one-half the pipe shell cut at its connection with the ring. The original

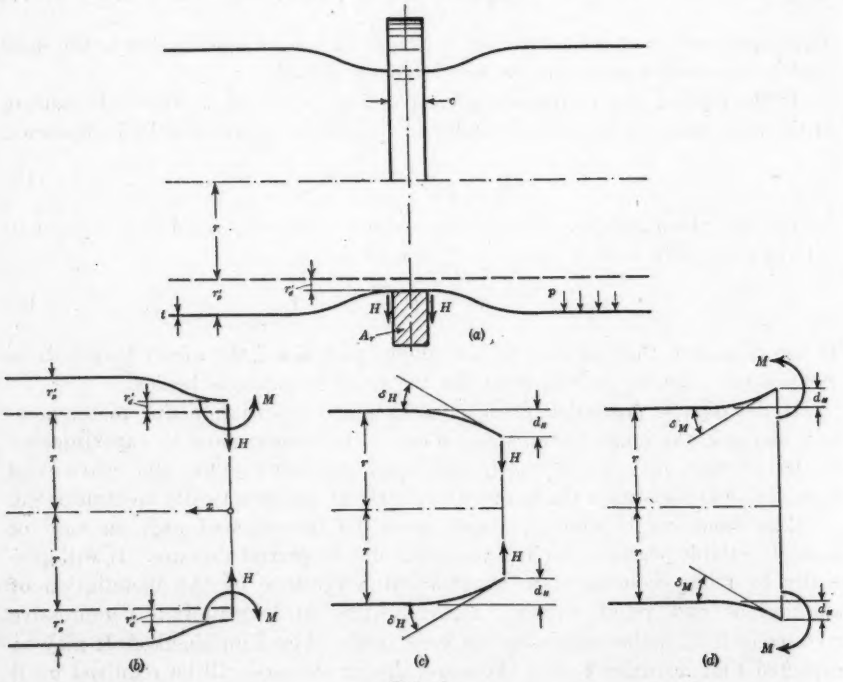

FIG. 4.

deformation of the free rim is restored by applying a uniformly distributed radial thrust, H, acting inward, and a uniformly distributed radial moment, M, which has the tendency to turn the rim outward. H and M are measured per unit length of circumference.

The rim deformations due to such radial loads are given by Meissner[6] and Pasternak.[7] The following analysis of the rim stresses is based on Pasternak's method.

If the free cylinder rim is loaded by the radial thrust, H, alone, as shown in Fig. 4 (*c*) the rim deformation is expressed by the radial deflection,

[6] "Zur Festigkeitsberechnung von Hochdruck-Kesseltrommeln," by E. Meissner, *Schweizerische Bauzeitung*, Vol. 86, July 4, 1925, p. 1.

[7] "Die praktische Berechnung biegefester Kugelschalen, kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischer Wandungen in gegenseitiger monolither Verbindung," by P. Pasternak, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, 1926, p. 1.

d_H, and the angular change of the generatrix, designated by δ_H. If the free rim is loaded by the radial moment, M, alone, as shown in Fig. 4 (d), the corresponding deformations are designated by d_M and δ_M, respectively.

Pasternak has derived the following expressions for the deformations of the rim, due to the radial loads, H and M:

$$d_H = \frac{2\,r^2\,H}{t\,z\,E} \qquad (14)$$

$$\delta_H = \frac{2\,r^2\,H}{t\,z^2\,E} \qquad (15)$$

$$d_M = \frac{2\,r^2\,M}{t\,z^2\,E} \qquad (16)$$

and,

$$\delta_M = \frac{4\,r^2\,M}{t\,z^3\,E} \qquad (17)$$

in which, z is a constant for the shell, equal to $\frac{\sqrt{r\,t}}{\sqrt[4]{3\,(1-m^2)}}$.

Due to the inside pressure, p, per unit area, the radius of the unrestricted shell portions increases by the amount,

$$r'_p = \frac{p\,r^2}{t\,E} \qquad (18)$$

The disk ring is loaded by the direct pressure, p, acting over the width, c, and the radial thrust, H, transmitted from both shell rims. Designating by N_t the total tension in the ring caused by the combined loads, then,

$$N_t = (p\,c + 2\,H)\,r \qquad (19)$$

Furthermore, if A_r designates the cross-sectional area of the ring,

$$r'_d = (p\,c + 2\,H)\,\frac{r^2}{A_r\,E} \qquad (20)$$

With reference to Fig. 4 (b), the following relation can next be written,

$$r'_p - d_H + d_M = r'_d \qquad (21)$$

Since the connection between the ring and the shell is assumed as fixed,

$$\delta_H = \delta_M \qquad (21a)$$

Substituting the corresponding values for the deformations, Equations (14), (15) (16), (17), (18), and (20), in Equations (21) and (21a), respectively,

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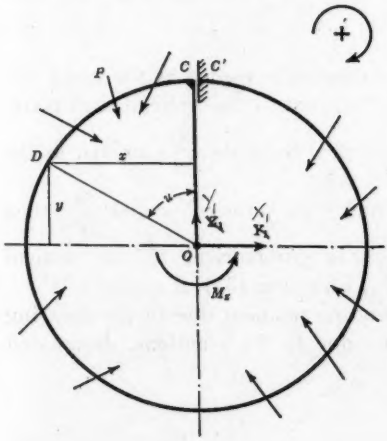

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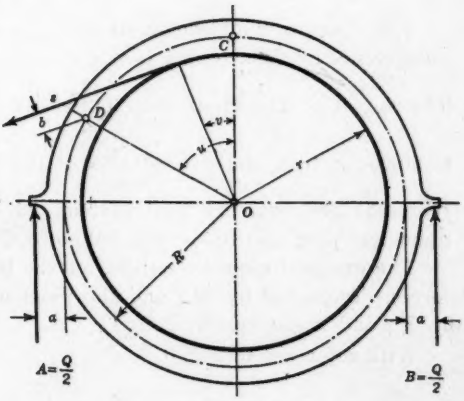

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$$X_1 = \frac{\int M_e\, y\, dl}{\int y^2\, dl} \qquad (31)$$

$$Y_1 = \frac{\int M_e\, x\, dl}{\int x_1^2\, dl} \qquad (32)$$

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The forces, X_1 and Y_1, are assumed positive when acting in the direction shown in Fig. 5, and all moments are taken as positive when acting clockwise.

Substituting $dl = R\,du$; $x = R \sin u$; and $y = R \cos u$; and integrating between the limits, $u = 0$ and $u = 2\pi$, Equations (31), (32), and (33) become,

$$X_1 = \frac{1}{\pi R} \int_0^{2\pi} M_e \cos u \, du \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

$$Y_1 = \frac{1}{\pi R} \int_0^{2\pi} M_e \sin u \, du \ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

and,

$$M_s = \frac{-1}{2\pi} \int_0^{2\pi} M_e \, du \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Designating by Q the total load of the pipe shell, transmitted by shear to one ring disk, the distribution of the shear, s, per unit width of circumference is given by Equations (5) or (9); therefore,

$$s = \frac{Q}{\pi r} \sin u \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Fig. 6 shows the circular ring disk, symmetrically loaded at the inner circumference by the shearing forces, s, acting tangent to the circle at each point. The ring is held in equilibrium by the two vertical reactions, $\frac{Q}{2}$, applied at the distance, a, from the neutral axis of the ring. The forces, s and $\frac{Q}{2}$, then represent the external load system, which is symmetrical to the vertical diameter. For this reason the unknown, Y_1, Equation (35), is zero.

The external moment consists of the bending moment due to the shearing forces, designated by M_s, and the moments due to the reactions, designated by M_A and M_B, respectively.

With reference to Fig. 6,

$$M_s = \int_0^u b\, s\, r \, dv \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

From analytic geometry,

$$b = R\left(\sin u \sin v + \cos u \cos v - \frac{r}{R}\right) \ldots\ldots\ldots\ldots\ldots (39)$$

and, therefore, with s from Equation (37),

$$M_s = \frac{Q\,R}{\pi} \int_0^u \sin v \left(\sin u \sin v + \cos u \cos v - \frac{r}{R}\right) dv \ldots\ldots (40)$$

By integration,

$$M_s = \frac{Q\,R}{\pi}\left[\frac{u}{2} \sin u - \frac{r}{R}\,(1 - \cos u)\right] \ldots\ldots\ldots\ldots\ldots (41)$$

The left reaction causes a moment between $u = \frac{\pi}{2}$ and $u = 2\pi$, equal to

$$M_A = \frac{Q\,R}{2}\left(\frac{a}{R} - \sin u + 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and the right reaction between $u = \frac{3\pi}{2}$ and $u = 2\pi$, causes a moment,

$$M_B = -\frac{Q\,R}{2}\left(\frac{a}{R} + \sin u + 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Equation (34) then becomes,

$$X_1 = \frac{1}{\pi R}\left[\int_0^{2\pi} M_s \cos u\, du + \int_{\frac{\pi}{2}}^{2\pi} M_A \cos u\, du + \int_{\frac{3\pi}{2}}^{2\pi} M_B \cos u\, du\right]. (44)$$

Substituting Equations (41), (42), and (43) in Equation (44), and integrating between the given limits,

$$X_1 = \frac{Q}{\pi}\left(\frac{r}{R} - \frac{a}{R} - \frac{3}{4}\right) \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For the moment, M_z, Equation (36) gives,

$$M_z = -\frac{1}{2\pi}\left[\int_0^{2\pi} M_s\, du + \int_{\frac{\pi}{2}}^{2\pi} M_A\, du + \int_{\frac{3\pi}{2}}^{2\pi} M_B\, du\right] \ldots\ldots (46)$$

Substituting Equations (41), (42), and (43), and integrating between the limits shown,

$$M_z = \frac{Q\,R}{\pi}\left[\frac{r}{R} - \frac{\pi}{4}\left(1 + \frac{a}{R}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

The resulting moment at any point of the ring is obtained by combining the external moment, M_e, with the moment, M_z, and the moment due to X_1. Designating the moment at the top of the ring, $u = 0$, by M_0;

$$M_0 = M_z - R\,X_1 = \frac{Q\,R}{\pi}\left[\frac{3}{4} - \frac{\pi}{4} + \frac{a}{R}\left(1 - \frac{\pi}{4}\right)\right] \ldots\ldots (48)$$

The normal force at this point, designated by N_0, is equal to X_1,

$$N_0 = \frac{Q}{\pi}\left(\frac{r}{R} - \frac{a}{R} - \frac{3}{4}\right) \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

Designating the resulting ring moment in the first quadrant of M_1, then, from Equations (41), (48), and (49),

$$M_1 = M_0 + N_0\,R\,(1 - \cos u) + M_z \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

or,

$$M_1 = \frac{Q\,R}{2\pi}\left[u \sin u + \frac{3}{2}\cos u - \frac{\pi}{2} + \frac{2\,a}{R}\left(\cos u - \frac{\pi}{4}\right)\right] \ldots\ldots (51)$$

Designating the resulting ring moment in the second quadrant by M_2, then

$$M_2 = M_0 + N_0\,R\,(1 - \cos u) + M_s + M_A \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

and, with reference to Equation (42),

$$M_2 = -\frac{Q\,R}{2\pi}\left[(\pi - u)\sin u - \frac{3}{2}\cos u - \frac{\pi}{2} - \frac{2\,a}{R}\left(\cos u + \frac{\pi}{4}\right)\right] (53)$$

A comparison of Equations (51) and (53) shows that the absolute moment values in both quadrants are the same. Except for the opposite sign, the moment curves are symmetrical about the horizontal diameter. On account of

symmetry in loading the moments in the third and fourth quadrant are identical with those in the second and first quadrant.

In order to determine the maximum moments it will be sufficient, therefore, to consider the first quadrant only. Extreme values occur at $u = 0$, at $u = \frac{\pi}{2}$, and at some intermediate angle, which is found by differentiating Equation (51) with respect to u, thus,

$$u \cot u = \frac{1}{2} + \frac{2\,a}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

The maximum intermediate moment, designated by M_i is obtained by substituting the value of u from Equation (51). Designating the moment for $u = \frac{\pi}{2}$ by M_a, Equation (51) gives,

$$M_a = -\frac{Q\,R}{4} \times \frac{a}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

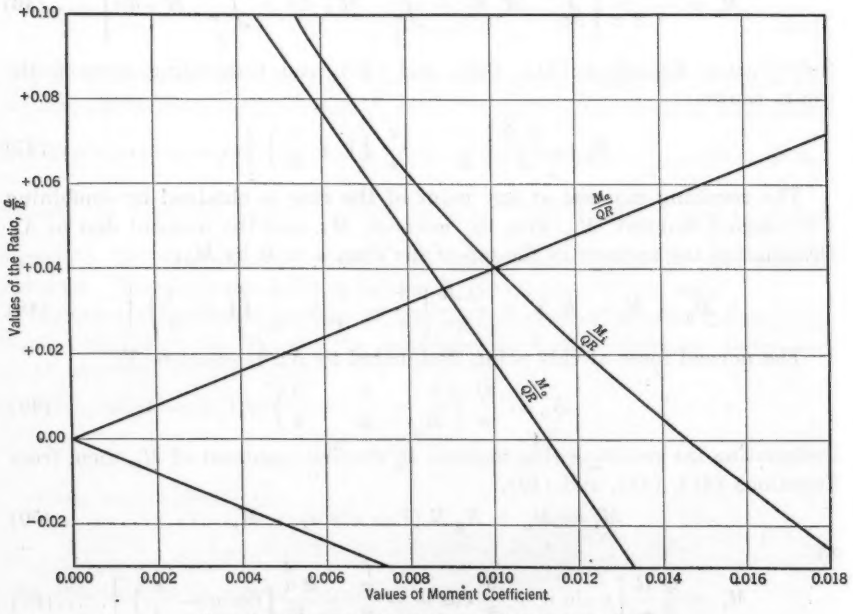

FIG. 7.—BENDING MOMENTS IN SUPPORTING RING.

Fig. 7 shows the absolute values of M_o, M_i, and M_a plotted in the function of the ratio, $\frac{a}{R}$. The eccentricity, a, is taken as positive when the reactions are applied outside the neutral axis of the supporting ring. The minimum possible ring moment is obtained for $\frac{a}{R} = +\,0.04$, in which case,

$$M_i = M_a = 0.010\ Q\ R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Fig. 8 illustrates the moment distribution for this case.

Karlsson,[11] by a different method, has derived the maximum ring moment for the special case, $a = 0$. He also quotes the absolute minimum moment derived by Hoeckerberg.[12] Fig. 7 checks both these values.

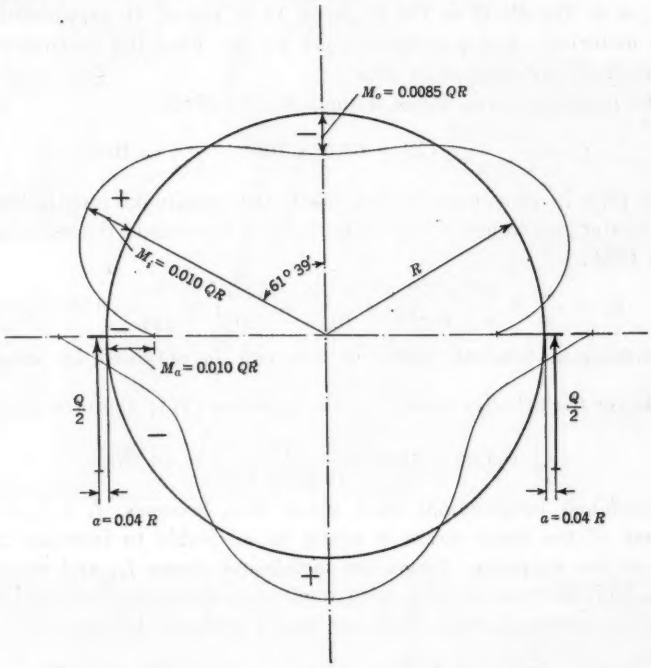

FIG. 8.—BENDING MOMENTS IN SUPPORTING RING FOR $\frac{\alpha}{R} = 0.04$.

The maximum normal ring force, due to the shearing forces, S, occurs at $u = \frac{\pi}{2}$ and is designated by N_a, expressed by the relation,

$$N_a = \frac{Q}{4} \qquad (57)$$

This force exerts compression just above, and tension just below, the horizontal diameter. Since, for $\frac{a}{R} \geqq +0.04$, the maximum bending stress is also found at this point, a somewhat smaller combined stress will be obtained with $+0.04 > \frac{a}{R} > 0.00$.

In addition to the normal force, N_a, the ring is stressed by the tensile force, N_t, Equation (30), which is independent of $\frac{a}{R}$, but can assume large values for higher heads in combination with thicker shells.

[11] "Ueber Schwerkraftspannungen in Rohrleitungen von grossen Durchmessern und deren rationelle Konstruktion," by Karl I. Karlsson, *Schweizersche Bauzeitung*, Vol. 80, September 2, 1922.

[12] *Teknisk Tidskrift*, 1919.

Example and Applications

Assume a pipe line, continuously supported by circular rings, and having the following characteristics: $D = 10$ ft.; $t = \frac{1}{4}$ in.; $L = 60$ ft.; $A_r = 1$ in. by 12 in.; $a = 0.04\ R$; $H = 100$ ft.; $w = 12$ lb. per sq. ft. (estimated, including joint material); and $q = 62.5$ lb. per cu. ft. Find the maximum stresses in the pipe shell and supporting ring.

For the maximum ring stress, Equation (12) gives,

$$f_r = \frac{10 \times 12}{2 \times 0.25}(12 + 62.5 \times 100) \times \frac{1}{144} = 10\,440$$

Since the pipe is continuously supported, the maximum longitudinal beam stress occurs at the supports; its magnitude is two-thirds the value given by Equation (13):

$$f_L = \frac{2}{3} \times \frac{60^2 \times 12}{4 \times 0.25}\left(\frac{2 \times 12}{10} + \frac{62.5}{2}\right) \times \frac{1}{144} = 6\,730$$

The maximum bending stress at the rim is obtained by substituting $f_r = 10\,440$ for the cylinder stress, $\frac{p\,r}{t}$, in Equation (27); therefore,

$$f_{bo} = 1.82 \times 10\,440\ \frac{12}{12\,25 + 1.51} = 16\,580$$

The combined longitudinal shell stress then becomes, $f_L + f_{bo} = 23\,310$. On account of the large stress it would be advisable to increase the shell thickness at the supports. Since the membrane stress, f_L, and the bending stress, f_{bo}, both decrease rapidly with increasing distances from the supporting disk, the increased plate thickness would probably be required for only one course.

The total reactions, in pounds, transmitted to one supporting ring is, $Q = 60\ (\pi \times 10 \times 12 + \pi \times 5^2 \times 62.5) = 317\,000$. Fig. 7, with $\frac{a}{R} = 0.04$, gives for the maximum ring moment, in inch-pounds, $M = 0.010 \times 317\,000 \times 66.25 = 210\,000$. The maximum bending stress in the supporting ring then is, $f_1 = \frac{210\,000 \times 6}{12^2} = 8\,750$.

From Equation (57) the maximum normal ring stress due to the shearing forces, s', equals $f_2 = \frac{317\,000}{4 \times 12} = 6\,610$. The normal ring stress due to radial forces, from Equation (30), equals $f_3 = \frac{43.4 \times 60}{16}\left(1 + 6.04 \times \frac{12}{12.25 + 1.51}\right) = 1\,020$. Since all these stresses occur combined at the horizontal diameter, the maximum fiber stress, tension, in the supporting ring becomes, $f = f_1 + f_2 + f_3 = 16\,380$.

Although principally developed for large pipe lines, the preceding analysis, with due modifications, may also be used for the design of horizontal storage tanks, tank cars, flumes, and under-water tubes supported on piers, etc.

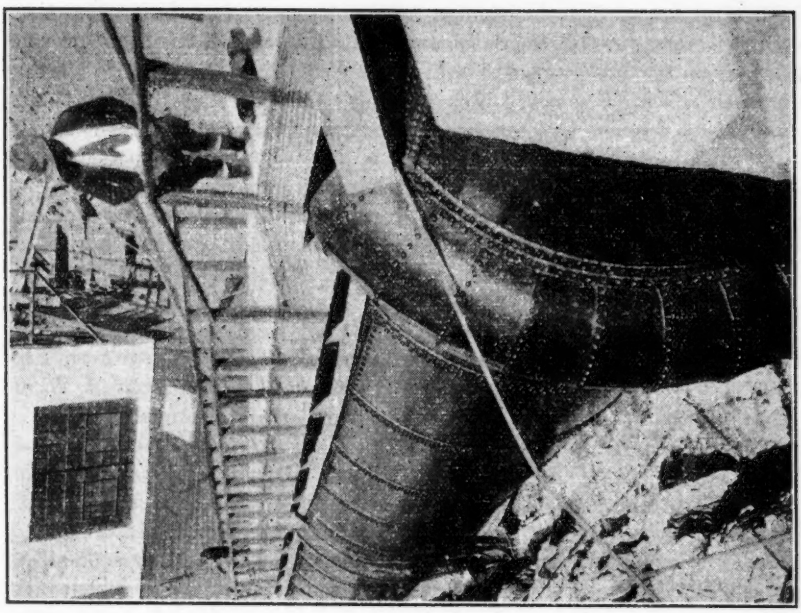

FIG. 10.—GLINES CANYON PENSTOCK PIPE.

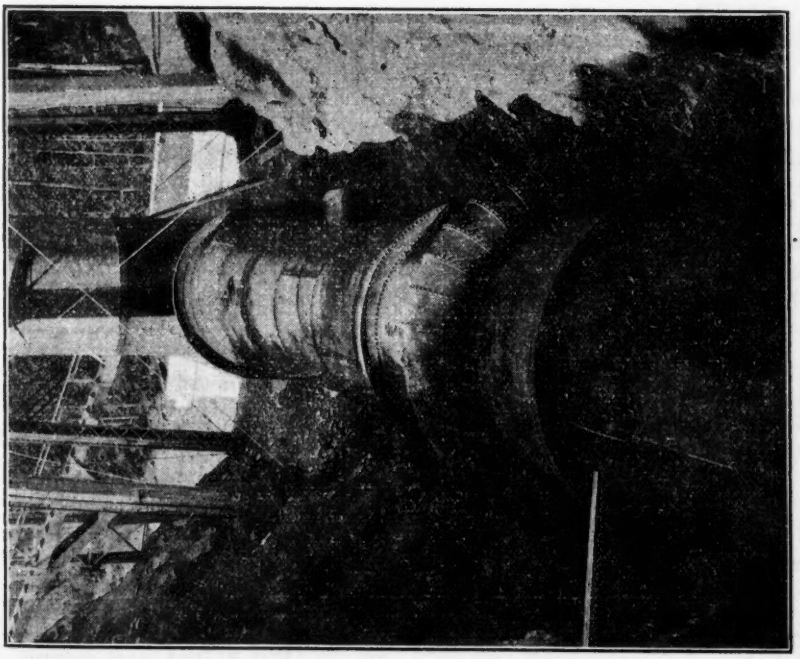

FIG. 9.—GLINES CANYON PRESSURE PIPE.

A horizontal tank, consisting of a cylindrical shell closed at both ends, may be supported by two stiff ring disks spaced so that true cantilever action gives a minimum longitudinal stress. By fixing one support and providing the other with roller bearings the structure becomes statically determinate, which condition eliminates temperature stresses and stresses due to yielding of the supports.

The design of self-supporting, semi-circular flumes is closely related to the theory of the Zeiss-Dywidag barrel-type roof. In case of reinforced concrete construction, the principal stress trajectories will indicate the most economical direction of the reinforcing bars.

Figs. 9 and 10 are views of the Glines Canyon pressure pipe and penstock, designed and built for the Northwestern Power and Light Company at Port Angeles, Wash. A general description of this project has been given[13] by W. B. McMillan, M. Am. Soc. C. E.

The horizontal pressure pipe has a length of 320 ft., of which about 70% is in a tunnel. The pipe has a diameter of 10 ft., a plate thickness of $\frac{7}{16}$ in., and operates under a static head of 120 ft. above the bottom of the pipe. The normal span is 46.5 ft., except at the expansion joint, shown in Fig. 9.

The penstock pipe is inclined at an angle of 34.5° to the horizontal and has a diameter of 11.5 ft., a plate thickness of $\frac{1}{2}$ to $\frac{3}{4}$ in., and a length of about 140 ft. between surge tank and lower anchor. Conservative design stresses in both pipes were adopted on account of water-hammer action.

The supporting rings were built up of plates and angles. Cast-steel rocker bearings, giving a low friction value, were used to transmit the load to concrete footings. There are no stiffening angles between the supporting rings.

The pressure pipe and the penstock pipe are each equipped with one expansion joint, consisting of a semi-annular ring of riveted construction, as shown in Figs. 9 and 10. This type of joint is adapted for fairly high pressures. In future installations, electric butt-welding of the joint plates is contemplated, which will give a more economical joint combined with increased flexibility.

Observations made on the empty pipe by M. Juul Hvorslev, Assoc. M. Am. Soc. C. E., show that the temperature variation at the top is considerably larger than at the bottom, which causes a differential movement between the upper and lower part, as confirmed by the measured deflections at the expansion joint.

Conclusions

The writer has reached six major conclusions as follows:

1.—Cylindrical shells, subject to water loads, are advantageously designed in accordance with the theory of thin shells of revolution, which permits an exact knowledge of the stress distribution and leads to a safer and more economical design than the customary empirical methods.

2.—If the pipe is supported by stiff, disk-shaped members, the pipe walls are mainly subject to direct stresses, except for a very narrow zone of bending stresses at the supports. For this reason the shell is entirely stable without any

[13] "The Glines Canyon Hydro-Electric Development on the Elwha River of the Northwestern Power & Light Co., Port Angeles, Washington," by W. B. McMillan, *Western Construction News*, April 10, 1927.

intermediate stiffening rings, thus permitting the use of large spans in combination with thin plates.

3.—On account of the reduced number of supports the use of bearings with small friction values becomes feasible. In connection with expansion joints it is possible practically to eliminate temperature stresses.

4.—The larger spans permit easy access for painting, and, in many cases, the cost of excavation can be considerably reduced.

5.—Due to the small deformations under all conditions of loading the joints remain tight.

6.—Pipe lines thus designed present a pleasant appearance, because the finished structure expresses the typical properties of a thin shell.

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PAPERS

HIGHWAY CONSTRUCTION MANAGEMENT[1]

By T. Warren Allen[2], M. Am. Soc. C. E.

Synopsis

The aim of construction management is to conduct operations properly in the least expensive way, whether the task be to construct a bridge, lay a pavement, or move cubic yards of earth.

In its broadest sense such management should begin with the consideration of the economic necessity and priority of procedure. Thus: Should the highway, A-B, be improved? What valid reasons are there for its improvement? What adequate return will there be for the expenditure that may be made? Should this road be improved prior to the improvement of the highway, C-D? What valid reasons are there for its improvement in advance of other highways that should also be improved? To avoid waste, an expenditure of public funds should await suitable answers to these questions. This paper describes how some of these problems have been approached by the Division of Management of the United States Bureau of Public Roads.

The Influence of Administration

Consciously or unconsciously, every administrator having charge of highway improvements, every locator, and every designer is frequently confronted with problems of construction management. During the 1929 construction season the writer encountered two outstanding illustrative cases. One was the construction of a part of a route out of turn. The location of this section was such that it could neither be used to local advantage nor to forward the construction of the remainder of the route. Thus, a sum of several hundred thousands of dollars was tied up, yielding no returns toward paying its interest cost for a period of nearly two years. Furthermore, had an adjoining section been constructed first it would have provided an improved road instead of an unimproved one for transporting the materials used in the first section.

[1] Presented at the meeting of the Highway Division, New York, N. Y., January 16, 1930. Written discussion on this paper will be closed in December, 1931, *Proceedings*.

[2] Chf., Div. of Management, U. S. Bureau of Public Roads, Washington, D. C.

The other case was that of a highway under contract to be paved. The route covered was mostly new location and several new bridges were required. These bridges were not included in the paving contract; they were let as a separate contract, but not until the paving work was nearly completed. As a result, the paving contractor had to move his plant around each stream, crossing by various detours and often at considerable expense. Incidentally, the paving contract also included the placing of the pavement over these bridges, which could not be done until long after the regular paving had been completed. The loss due to the failure to complete the bridges at an earlier date extends much farther than the mere cost of moving the plant around the detours. For example, while the contractor, in the course of seven days, had to move his equipment to six new locations to dodge unbridged streams, the length of haul from move to move varied as much as 4 miles. Thus, one day it was 3.2 miles; the next it was 7 miles, and the following day, 3 miles. It was impossible to keep a properly balanced hauling equipment on hand, and the rate of production was naturally low even for the actual working time.

These illustrations show the influence of administration on construction management. Another example, concerns the mixing time of concrete for the pavement slab. State highway specifications for the time the concrete materials must be held in the drum during mixing vary from 60 to 105 sec. Since modern paving machines require about 15 sec. additional for charging and discharging, it follows that a 60-sec. mixing requirement permits of turning out a maximum of 48 batches per hour, but a 105-sec. specification reduces the maximum possible output to 30 batches per hour. Bearing in mind that the daily operating cost is practically the same in both cases, it is evident that in the latter case the cost of production is increased tremendously over what it would be for the former. The mixing action is not changed by merely moving the paver across State lines. If 60 sec. of mixing produces concrete of the required strength and quality in one State, why is this not also true in a neighboring State? From another viewpoint, if quality is improved by a longer mixing time, is it, for example, improved sufficiently by 45 sec. of additional mixing to pay the cost of 37½% reduction in the rate of production?

Highway Management Research

In this connection Table 1 shows typical averages of results of more than three years' work by the Division of Management of the Bureau of Public Roads, which seem conclusive. It gives the average breaking strength of standard cylinders made from samples taken, respectively, from each corner and from the center of the batch as dumped on the sub-grade when the mixing time was varied from 45 to 75 sec. Each entry represents an average of from five to ten cylinders. Cylinders *A* and *B* were taken, respectively, from the left and right-hand corners farthest from the mixer, Cylinders *C* and *D*, from the right and left-hand corners nearest the mixer, and Cylinder

E, from the center of the batch as dumped on the sub-grade. The locations of the jobs given by numbers are, as follows:

Job No.	Location
1	Mahoning County, Ohio
2	Miami County, Ohio
3	Coshocton County, Ohio
4	Crawford County, Arkansas
5	Gadsden County, Florida
6	Leon County, Florida
7	Harnett County, North Carolina

TABLE 1.—EFFECT OF MIXING TIME ON STRENGTH OF CONCRETE

Job No.	Mixing time, in seconds	Part of Batch From Which Samples Were Taken					Job averages
		A	*B*	*C*	*D*	*E*	
1	45	4496	4710	3960	4568	4428	4432
2		4968	4715	4753	4780	4724	4788
3		4727	4900	5108	4994	4961	4938
4		4670	4562	5205	4744	4941	4829
5		3950	4028	3985	3519	3988	3894
6		3664	3845	3977	3924	4120	3905
7		3123	2803	3212	3381	3370	3177
Averages		4228	4223	4314	4272	4362	4280
1	60	4210	4314	4578	4406	4038	4309
2		4826	5092	5078	4955	5084	5007
3		4967	5015	5081	4942	5028	5007
4		4174	3854	4000	4240	4174	4085
5		4327	4243	4124	4141	4123	4191
6		3404	3183	3099	3168	3221	3215
7		3002	2500	2946	3084	3169	2948
Averages		4130	4028	4130	4134	4120	4109
1	75	4420	4596	4608	4254	4072	4390
2		4700	4683	4644	4636	4738	4680
3		4982	5065	5026	4843	4927	4969
4*		4425	4140	4310	4604	4822	4260
5		4188	4180	4236	3941	4007	4110
6		3972	4002	3888	4188	4137	4037
7		3456	3278	3152	3210	3314	3282
Averages		4306	4277	4266	4242	4145	4247
General averages		4221	4176	4237	4216	4209	4212

* This series was mixed in 90 sec.

Fig. 1 also shows graphically the result of more than 3 000 individual tests made on about forty different highway construction projects under a great variety of actual field conditions. These show definitely that the modern road paver in good condition will produce a high quality of concrete in 45 sec. and that certainly nothing of material value is to be gained in strength by extending the mixing time beyond 60 sec.

The field tests also show that with modern equipment and alert management, operation can be maintained on the shorter mixing time at as high a degree of efficiency as on longer mixing times. Furthermore, this additional production can be secured with the employment of little, if any, additional labor except the necessary additional trucks and drivers to deliver the additional batches.

The influence of design on construction management and costs is far-reaching. In the case of a bridge, for example, the design largely controls the methods which can be used in both the fabrication and the erection. In the case of the highway itself the design usually includes many items which vary widely, depending on whether the designer gives real thought to his work and is familiar with the construction problems involved or merely follows the lines of least resistance. The average designer in dealing with the item of grading, for instance, is quite careful to balance his cuts and fills, but he usually fails to give sufficient consideration to the lengths of haul he imposes or how widely or frequently these lengths may fluctuate. Inattention to these details makes it very difficult, if not impossible, for the contractor to adjust the size of his hauling outfit properly so as to fit the work to be done, and increases the cost materially.

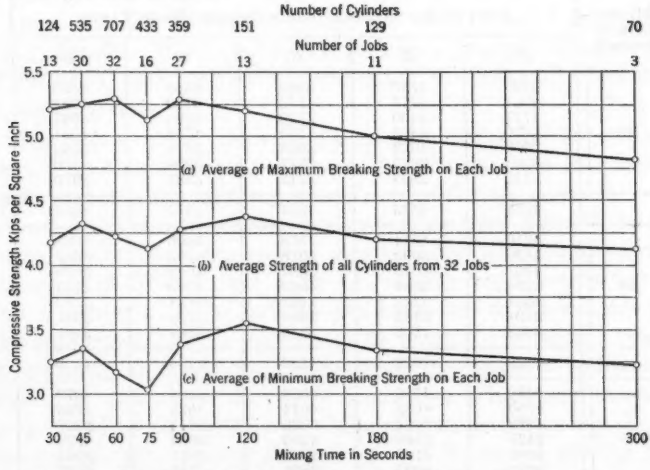
Number of Cylinders
124 535 707 433 359 151 129 70
Number of Jobs
13 30 32 16 27 13 11 3
5.5
5.0
4.5
4.0
3.5
3.0
Compressive Strength Kips per Square Inch
(a) Average of Maximum Breaking Strength on Each Job
(b) Average Strength of all Cylinders from 32 Jobs
(c) Average of Minimum Breaking Strength on Each Job
30 45 60 75 90 120 180 300
Mixing Time in Seconds

FIG. 1.—EFFECT OF MIXING TIME ON STRENGTH OF CONCRETE.

This may be better appreciated by reference to the actual variations in haul from day to day encountered on a power-shovel grading project. A few consecutive days selected at random from this project show the following average daily haul lengths, in feet; 900, 400, 200, 1 000, 1 000, 550, 800, 1 000, 350, 1 100, and 650.

In this case assume that the shovel was excavating at the rate of 750 cu. yd. of common excavation per 10-hour day. Then, for example, if 1.5-yd. wagons, traveling at a speed of 240 ft. per min., were used for hauling, the number of wagons required for each day would be 9, 5, 4, 10, 10, 6, 9, 10, 5, 11, and 7, respectively.[3] Under such conditions it is practically impossible for the contractor properly to adjust the size of his hauling equipment to the output of the shovel. The condition is really worse than shown, because quite wide variations in hauling lengths also occurred during the progress of each day's work.

[3] A full discussion of the method of determining the number of wagons may be found in *Public Roads*, March, 1928, and for the number of trucks in paving work, in *Public Roads*, February, 1931.

As a matter of fact in a case like this, the contractor being unable to vary his hauling equipment in such a zigzag fashion, usually decides upon a fixed number of hauling units, say eight, for illustration. He then finds that for the first day's work he is one wagon short, and as the shovel can only dig when there is hauling equipment available the output for the day is 83⅓ cu. yd. short, and at, say, 60 cents per cu. yd., this means a loss of $50 minus $10 (estimated cost of one wagon, including driver), or $40. The following day there is a surplus of hauling equipment of three wagons which at $10 per day means a loss of $30. In like manner, for the succeeding days losses occur amounting to $40, $70, $20, $40, $70, $30, $92.73, and $10, respectively, or an average loss of about $46.63 per day.

The foregoing discussion is introduced solely to bring to the attention of highway engineers matters which if given due consideration should operate not only to help in reducing the cost of highway improvement, but also to raise the standard of the profession.

Studies of the highway constructor's methods and costs made by the Division of Management, U. S. Bureau of Public Roads, have been in progress for several years and while they have been confined to highways, it is believed the principles developed should be servicable in other lines of construction.

A full description of the endeavors in all fields of highway contracting would extend the length of this paper unduly. The Bureau of Public Roads has studied grading in both common and rock excavation done with various kinds of equipment, as well as steel and concrete bridges, and many types of surfacing. Incidentally, it has also studied the operation of the equipment used and has given the individual manufacturers concerned the benefit of its findings. For example, manufacturers welcome photographs and descriptions of breaks that occur, or the rate of wear on mixer buckets. The latter is a factor, not only in the thoroughness of mixing, but excessive wear seriously retards the discharge time of the batch.

Analysis of Time Losses

In general, the method used has been to make stop-watch analyses of the various activities on many going projects, covering practically all classes of road work. By this means average performance data have been secured for construction work both by districts and for the entire country. Of course, data showing excellent performance have also been secured. From this experience, and with these averages as a measure, the performance of the individual constructor as determined by stop-watch readings can be analyzed so as to show him not only wherein he fails to attain the fruits of average or excellent management, as the case may be, but also to indicate to him how his future performance can be improved.

The studies invariably disclose time losses. These are divided into major losses (which are of 15 min., or more, in duration), and minor losses, which are of less than 15 min. duration. Major and minor losses are then usually each subdivided into Classes A and B. Class A, in general, includes all

those arising from the action of the elements and from contingencies which the exercise of good managerial ability would not have been expected to anticipate and prevent; and Class B includes all time losses which good management, under the conditions of the particular job in question, should have found possible to eliminate. The degree to which a contractor profitably eliminates the Class B losses indicates his grade of management. The results of these studies are recorded on a weekly report known as "Form C. S. 13" (see Table 2). A copy of this form properly filled out is given to the contractor whose job is being studied.

Its purpose is to bring together the data collected in regular production studies into a concise and compact form so as to emphasize as clearly and forcibly as possible the extent or amount of the various classes of time losses which are being incurred, and the effect that these time losses (which seem possible of elimination) are having on the rate of production.

The major time losses consisting of definite stops involving several minutes or hours in duration are, of course, fully apparent and every contractor makes efforts to prevent their occurrence or recurrence. The minor time losses, which are largely repetitive and involve only seconds, are not so apparent to the management; and yet on the average road job, as determined from studies on many hundreds of going projects, the minor time losses consume from 20 to 50% of the total time that the crew is at work. Furthermore, definite stops of from 15 min. each to several days in length consume from 25 to 50% of the otherwise available calendar days. In other words, on the average job about one-half the theoretically available calendar working time is a total loss in so far as any useful production is concerned.

On the basis of studies of nearly a hundred concrete paving jobs with modern equipment, fairly well distributed over the entire United States, the Bureau of Public Roads has found that, on what might be called the average concrete paving job, after the contract is secured and the plant and crew assembled, the mixer is only used for pouring concrete about 60% of the available calendar working hours. The remainder of the time is lost from the following causes:

MAJOR TIME LOSSES

	Percentage
Rain	9.5
Wet sub-grade	8
Lack of prepared sub-grade	3
Lack of materials	3.5
Moving plant set-up	3.5
Inadequate supply and faulty operation of hauling equipment	3
Mixer trouble	2
Lack of water at mixer	2
Loading plant trouble	1.5
Miscellaneous causes	4
Total	40

TABLE 2.—SUMMARY OF TIME LOSSES AND THEIR EFFECT ON PRODUCTION (FORM C. S. 13).

State_____ County_____ F. A. P. No._____ Project No._____

Type of project_____ Dates, including or week ending_____

Major equipment_____ Auxiliary equipment_____

Total available working time, hours_____ Total estimated production_____

Total major delays occurring during available working time — *Character*	Class A		Class B		Total all major delays	
	Hours	Per cent	Hours	Per cent	Hours	%
Totals						
Time major equipment actually operated						

Total minor delays occurring during time of actual operation, as extended from stop-watch studies — *Character*	Class A		Class B		Total all minor delays	
	Hours	Per cent	Hours	Per cent	Hours	%
Totals						
Time major equipment operated at 100% efficiency						

Grand total all Class B time losses, hours.....

Possible operating time, all Class B losses eliminated, hrs.....

With all Class B losses eliminated, production would have been.....

Over-all efficiency of major equipment operation, %.....

NOTES.— "Major delays" include all stops of 15 min. or more in duration, while "minor delays" include all stops less than 15 min. each in duration as found by stop-watch studies and extended to time of actual operation. Use back of this form for Remarks. Give total extent of stop-watch studies, in hours, for week.

These definite stops, each of 15 min., or more, in duration, do not comprise the entire time loss or time during which the key equipment is idle. Stop-watch studies on these same jobs show that an average of nearly 16% of the total available time, or 26% of the time the crew is at work, is lost in minor stops and interruptions, each less than 15 min. in duration. These are due to the following causes:

MINOR TIME LOSSES

	Percentage
Supply and operation of hauling equipment	7.3
Lack of, or trouble with, water at mixer	1.9
Lack of materials and supplies in place	0.7
Mixer trouble	1.1
Mixer operator	1.3
Finishing	0.4
Sub-grade delays	1.6
Miscellaneous causes	1.3
Total	15.6

In other words, the average concrete road paving contractor is not only out on the road with his crew only about 60% of the available calendar hours, but during this time he is actually operating at only about 74% of the rate at which his key equipment is capable of maintaining. The details from which these averages are derived were assembled in forms, of which samples are illustrated by Tables 3 and 4.

Table 3 is arranged to show the time available, time lost, production, and stop-watch data observed on about eighty concrete paving jobs during 1926, 1927, 1928, and 1929. The code numbers in Columns (5) and (21) denote: 1, new; 2, very good, or excellent; 3, good; 4, fair; 5, poor; and 6, very poor. The distribution of time losses for various causes is recorded by the Bureau of Public Roads as illustrated in Table 4. The major losses, involving stops of 15 min. or more, each, are segregated in a record form identical with Table 4, except for some modifications in corresponding column headings. The totals and the column headings for this table, on the jobs listed in Tables 3 and 4 are, as follows:

Total time that should have been available, and the percentage of this time that was lost for various causes, in definite stops of 15 min., or more:

Column (3), total available time, in hours	33 309.72
Total time that the mixer was idle:	
Column (4), hours	13 342.46
Column (5), percentage of total available time	39.95

Causes from which time losses (in percentage of total time) were incurred:

Column (6), miscellaneous	2.63
Column (7), operation	0.47
Column (8), shortage	2.15
Column (9), moving mixer or plant set-up	3.62
Column (10), mixer trouble	2.04
Column (11), lack of materials at yard	3.55
Column (12), trouble at batcher plant	1.47

Column (13), lack of, or trouble with, water	2.15
Column (14), finishing	0.50
Column (15), wet sub-grade	7.29
Columns (16) and (17), joints, or steel	0.10
Column (18), cold	1.05
Column (19), sub-grade not prepared	3.37
Column (20), rain	9.56

In road grading with the power shovel, much the same condition is found as in paving, except that the weather losses are much smaller and the minor or short-period interruptions or delays consume a much larger proportion of the working time. Thus, on the basis of more than 100 power-shovel, road-grading jobs, the Bureau of Public Roads has found that on the average, after the contract is secured and the equipment is on the ground, the major losses are about 30% of the total available calendar hours, these being in definite stops each of 15 min., or more, in duration. Such major delays are due to the following causes (in percentage of total losses):

Rain and wet ground	9.4
Repairs to shovel and other equipment	10.5
Moving from cut to cut	2.8
Drilling and blasting	2.1
Hauling equipment trouble, or shortage	1.4
Taking on, or lack of, fuel or water	0.9
Miscellaneous causes of various kinds	3.0
Total	30.1

Stop-watch studies also show that about 27% of the total available calendar hours, or about 39% of the time the shovel is supposedly at work, is unproductive. These minor delays or interruptions to continuous operation are due to the following causes (in percentage of total available calendar-hours):

Lack of sufficient hauling equipment	8.2
Improper operation of hauling equipment	5.4
Moving shovel within cut	3.3
Checking grade	0.3
Shovel repairs, minor	2.3
Handling large rocks, boulders, stumps, etc	2.5
Blasting boulders, rocks, etc	0.4
Taking on fuel	0.7
Operator	1.1
Miscellaneous causes of various kinds	2.9
Total	27.1

These figures are averages of a great many jobs and must not be confused with the performance on individual jobs. Weather and other uncontrollable causes often affect one job seriously, but may leave another practically untouched. Thus, the Bureau of Public Roads has found concrete paving jobs which have operated for a month or more at a time loss not exceeding 10% of the available calendar hours and which utilized more than 90% of the time at work in useful production. This, however, is exceptional and prob-

TABLE 3.—JOB RECORDS SUMMARY OF CONCRETE PAVING WORK. TIME AVAILABLE, TIME LOST, PRODUCTION, AND STOP-WATCH DATA

Job location	Job No.	Paver			Mixing time required, 1929 standards, in seconds	Year study made	Stop-Watch Studies					Production Record for Period Job Was Under Observation						Hauling Equipment		
		Make, code	Size, E	Condition, code			Total time of study, in minutes	Total time lost, in minutes	Percentage of time mixer idle	Total batches during studies	Average rate of production batches per hour	Total hours expected to be available	Total time mixer idle, stops of 15 min. or more,	Percentage of time mixer idle	Total batches poured	Average rate of production, batches per hour	Average length of haul	Kind	Batches per load or train	Condition average, code
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
14-09-01	1	06	27	3	90	1928	3 878.84	744.7	19.2	1 941	30.0	403.00	108.00	26.8	8 635	29.3	4.2	Trucks...	4	2
21-11-01	2	10	27	1	75	1927	3 720.00	1 380.0	37.1	1 542	24.9	531.50	251.30	47.3	7 299	26.0	4.1	Trucks...	2	3
21-11-02	3	04	27	4	75	1927	2 400.00	918.5	38.3	921	23.0	434.50	236.30	54.4	4 015	20.2	0.5	Trucks...	3	2
23-06-02	6	04	27	2	75	1927	1 120.00	246.3	21.9	452	24.2	156.90	44.50	28.4	2 822	25.2	4.2	Trucks...	2	3
23-06-06	7	06	21	3	75	1927	1 260.00	192.9	15.3	640	30.5	191.75	62.23	32.5	3 851	29.8	2.1	Trucks...	2	2
23-06-09	8	09	21	5	75	1927	1 140.00	131.9	11.6	505	26.6	137.75	84.75	61.5	1 235	23.1	1.2	Trucks...	3	3

TABLE 4.—JOB RECORDS OF STOP-WATCH STUDIES ON CONCRETE PAVING WORK SHOWING TOTAL TIME OF STUDIES AND PERCENTAGE OF THIS TIME WHICH WAS LOST DUE TO VARIOUS CAUSES

Job location	Job No.	Total study time, in minutes	Total Time Lost		Causes From Which Time Losses (in Percentage of Total Time) Were Incurred													
			Seconds	Percentage of study time	Miscellaneous	Hauling Equipment		Dumping batches	Mixer trouble	Inspection	Mixer operator	Water supply at mixer	Finishing	Handling cement	Joints	Steel	Setting parting strip	Subgrade not prepared
						Operation	Supply											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
14-09-01.....	1	3 878.84	44 684	19.1	0.7	1.4	1.8	5.5	1.0	0.1	4.2	2.6	0.3	....	0.2	1.1	0.1	....
21-11-01.....	2	3 720.00	82 800	37.1	0.6	1.8	15.1	6.1	1.1	0.0	2.7	8.5	1.1	....	0.0	0.1	0.0	0.0
21-11-02.....	3	2 400.00	55 109	38.3	2.0	1.5	3.1	2.7	2.3	....	15.9	4.0	0.9	1.7	0.1	1.5	1.1	1.3
23-06-02.....	6	1 120.00	14 777	21.9	....	0.8	5.4	0.8	1.0	2.8	2.4	0.5	....	0.2	1.6	....	....	6.4
23-06-06.....	7	1 260.00	11 573	15.3	0.4	1.5	2.2	1.9	0.0	....	0.7	2.0	....	0.8	0.2	0.9	0.3	4.4
23-06-09.....	8	1 140.00	7 914	11.6	....	3.1	0.4	1.3	0.1	....	3.3	1.9	....	0.3	....	0.1	0.4	0.7

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ably could be expected to be attained only during the most favorable part of the construction season. The studies also indicate that as contractors become better aware of the losses incurred, the percentage of such losses decreases.

Method of Determining Required Number of Hauling Units

When hauling units are used in different types of road construction an immediate problem confronts the contractor as to the number of units required in hauling material. Usually, a contractor does not predetermine the proper number of trucks, wagons, or other typical hauling equipment that is necessary to handle the particular material which is involved. If it appears that he started with an insufficient supply he orders more, but not until an unnecessary amount of time and money has been lost.

In concrete road construction or similar work, trucks are usually hired and paid at so much for each batch or each batch-mile. This price is influenced by the length of the haul. If too many trucks are on the job they eliminate one another by the income they receive, or they are discharged by the contractor's superintendent. If not enough trucks are present, more are hired. It is only by careful planning and management on the part of the superintendent that a sufficient number of trucks are present to supply the mixer. This is true in other lines of construction where hauling units are employed.

In power-shovel grading where the haul is relatively long, large-sized hauling equipment is frequently used. In shorter hauls, teams and wagons, light trucks, and iron mules are often used. With the larger units extreme care must be taken when deciding on the proper number of hauling units. It may appear that an insufficient number of units is employed, yet, if another large-sized unit is added, it may not be economical; but whether large or small units are employed it is necessary to keep all of them busy. If they are not sub-contracted, it is necessary, if possible, to shift part of them to longer hauls when an over-supply of units is available. The foregoing statements concerning power-shovel grading may be applied also to the elevating grader.

In any construction it is absolutely essential to maintain a good haul road. After a rain the road is soft and becomes rutted and full of "chuck" holes. One or more men with the proper equipment should be assigned as rapidly as possible to the constant maintenance of the hauling road. Failure to observe these rather elementary requirements probably explains why the average contractor loses more money through shortage of hauling equipment than from any other controllable cause.

Efficiency in Production

These time losses would not be so serious if the cost of operation varied with the output. Unfortunately, however, the daily or hourly cost of operating almost any kind or type of modern well-equipped road-building outfits is nearly the same whether production is at only a fraction of its capacity

or at its maximum rate. In concrete road construction, for instance, the operating cost of a modern outfit, exclusive of materials, but including all labor, supplies, overhead, and rental value of the equipment, is seldom less than $40 per actual working hour and may readily exceed $50 per working hour. Frequently, however, a plant with an operating cost of, say, $45 per working hour and an actual capacity of 45 to 48 batches per hour is, at least at times, producing only 20 batches per hour. The operating cost is then $2.25 per batch, whereas capacity production would cut the cost to about $1.00 per batch.

It is thus quite plain that the contractor should do everything in his power to increase production. Frequently, the observation is made by engineers that it is no concern of the engineer whether the contractor makes or loses money. With but few exceptions, the cheapest method of construction is by contract, and a prosperous contracting fraternity is essential if work is to be well done. Increased efficiency in contract work will ultimately be reflected in lower bid prices. Probably without realizing it, highway contractors are bidding on a basis of doing work at a little more than 50% efficiency. Some of the contractors are doing a profitable business and some are not. The proper procedure for all of them, in order that they may do a more profitable business, should be more efficient management rather than increased prices.

The Bureau of Public Roads, therefore, believes it is working in a worthy cause and in a fertile field when it undertakes to assist highway contractors in increasing production. To this end the Bureau has met with gratifying success. In some few cases it has been possible nearly to double the rate of production with little or no addition to the cost of operation. These cases are exceptional, of course, but still fewer are the cases in which it has not been possible to show some improvement in the rate of production without entailing a corresponding increase in the cost of operation.

Having in mind the causes of low production, attention then logically turns to their elimination or reduction. Frequently, it is an easy matter to remedy some of the causes which have been most influential in keeping down the rate of production. For example, it is not uncommon to find that in power-shovel grading improper placing of the hauling units imposes an unnecessarily long boom-swing which increases the bucket cycle by as much as 5 sec. In good common excavation the elimination of this one delay alone may mean an increase of from 20 to 25% in the rate of production, an item surely worth while.

As a specific example of what was accomplished in the summer of 1929 by ascertaining losses and bringing their meaning to the attention of the contractor the following is given:

On a Portland cement concrete paving job 12.41 miles long and 18 ft. wide, with a thickness of 8 in., 6 in., and 8 in., the pavement was being laid during the summer months. Standard, modern equipment was used throughout, and production studies were conducted between August 19 and October 5. The daily record of production and time losses before these studies were

TABLE 5.—EXAMPLE OF SUMMARY SHEET OF WEEKLY REPORTS OF TIME LOSSES

State, Tennessee. County, Haywood. Project No., S. A. P. 358-A.

Type of project, Portland cement concrete. Duration of study, from August 19, 1929, to October 5, 1929. Weeks, 7.

Major equipment, 27-E paver. Auxiliary equipment, two-batch trucks.

Week ending, 1929	Production, in square yards	Total available working time, in hours	Major Delays			Minor Delays			Total of all delays, Column (6) + Column (9)	Time major equipment operated at 100% efficiency, Column (3) to Column (10)	Total of all Class B delays, Column (5) + Column (8)	Possible operating time eliminating Class B delays, Column (11) + Column (12)	Possible production eliminating Class B delays	Percentage of over-all efficiency, Column (11) Column (13)
			Class A, in hours	Class B, in hours	Total, in hours	Class A, in hours	Class B, in hours	Total, in hours						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Aug. 24....	10 981	68.0	1.75	3.15*	4.90	0.55	10.75*	11.30	16.20	51.80	13.90	65.70	13 910	78.8
Aug. 31....	12 052	68.0	2.00	2.43	4.43	0.00	11.00*	11.00	15.43	52.57	13.43	66.00	15 100	79.8
Sept. 7....	9 982	67.5	7.5‡	4.25	11.75	1.53	11.50*	13.03	24.78	42.72	15.75	58.47	13 700	72.8
Sept. 14....	8 422	68.0	23.08§	1.00	24.08	1.65	4.06*†	5.71	29.79	38.21	5.06	43.27	9 540	88.4
Sept. 21....	12 812	68.0	1.58	0.00	1.58	0.00	(12.53*)	12.53	14.11	53.89	12.53	66.42	15 800	81.2
Sept. 28....	10 426	68.0	17.50† ‖	0.00	17.50	1.08	4.33*	5.41	22.91	45.09	4.33	49.42	11 426	91.2
Oct. 5....	6 920	67.5	(33.58‡)	0.00	33.58	3.40	0.72	4.12	37.70	29.8	0.72	30.52	7 058	98.2
Total														

* Truck supply and operation delays comprise greater part of this total: September 21, 6.84 hours; September 28, 2.43 hours.

† Water supply delays comprise greater part of this total: September 21, 5.09 hours.

‡ Rain delays comprise greater part of this total.

§ Wet sub-grade delays comprise greater part of this total.

‖ Move pipe line delays comprise greater part of this total.

started, is not given. Paving operations had been in progress for one month prior to August 19 and during this time only 1.53 miles, or 16 157 sq. yd. of pavement were laid. The biggest day's run during this period was 330 batches, or 991 lin. ft. of pavement. On only one or two days did the outfit lay more than 800 lin. ft. of pavement. During the seven weeks that the study was in progress, 6.76 miles of pavement were laid. During the week ending September 21, 2 168 batches, or 6 406 lin. ft. of pavement were laid. The biggest day's run was 411 batches, or 1 226 lin. ft. of pavement in 11.5 hours. The summary in Table 5 shows that the efficiency of the job increased from 78.8% for the week ending August 24, to 88.4% for the week ending September 14, and to 98.2% for the week ending October 5 (see Column (15)).

Another example of increased production with unit costs decreased is shown in the results of studies made on Wisconsin Federal-Aid Project No. 460-A. This was a Portland cement concrete paving job, 14.7 miles long, in Dane and Columbia Counties. Studies were conducted between June 3 and August 24, 1929.

The contractor's major operations were analyzed and the cause and extent of all time losses determined. This information was furnished to the contractor who used it as a guide in his efforts to increase production. The trend of operations during the studies is shown in Table 6.

TABLE 6.—PRODUCTION STUDIES, FEDERAL-AID PROJECT NO. 460-A IN 1929

Item No.	Description	June	July	August
	DISTRIBUTION OF TIME LOSSES, IN PERCENTAGE OF TOTAL TIME SPENT			
1	Mixer (operator)	2.3	2.8	1.7
2	Mixer (mechanical)	1.4	1.2	1.2
3	Truck (operation)	4.6	4.0	3.7
4	Truck (supply)	13.0	7.6	1.6
5	Installing joints	0.9	4.0	2.1
6	Sub-grade	2.1	2.0	1.9
7	Water supply	2.4	1.4	2.2
8	Moving plates (track for trucks)	1.0	0.7	0.3
9	Materials	1.3	0.0	1.3
10	Miscellaneous	2.3	1.7	2.4
11	Dry batches	0.0	0.7	1.0
12	Total delays	31.3	26.1	19.4
	COSTS			
13	Cost per square yard	$1.44	$1.30	$1.26
14	Reduction, in percentage, of June costs	....	9.7	12.5
	MISCELLANEOUS DATA			
15	Number of days in which concrete was poured	24	23	15
16	Average length of working days, in hours	11	11.5	11.5
17	Total production, in square yards	36 636	40 285	30 381
18	Average production per day in which concrete was poured, in square yards	1 526	1 749	2 025

The greatest cause of lost time, as shown by the studies, was due to a shortage of hauling equipment. This was pointed out to the contractor early in the studies and, as a result, he realized the importance of supplying a sufficient number of hauling units for full mixer production. This delay

was materially reduced. The reduction in unit cost is due to the increase in production resulting from the elimination of delays as a result of the studies.

The foregoing are only a few of many studies in which production was materially increased without a proportionate increase in unit costs. It is not always possible to increase production at the rates shown in the examples given, but on the other hand failures to record some improvement are rare.

Conclusions

A general observation of the following rather elementary suggestions should do much to increase the efficiency with which the average road construction outfit is operating:

1.—The rate of production is so dependent on the performance of the key equipment as to demand that it always be maintained in good mechanical condition, and to forbid the employment of any other than a high-grade operator.

2.—Auxiliary or dependent equipment which is of insufficient capacity or which cannot maintain the rate demanded by the operations of the key equipment, should not be tolerated. This applies also to the operators of such equipment.

3.—The use of equipment or methods of operation which cannot readily be adapted to operating efficiently in connection with the assembly or outfit as a whole, or to sustain economically the production rate of the key equipment, should be avoided.

An assembly of men and equipment, no matter how fine, is practically worthless as a producing unit until supplied with a directing head, because on the managerial ability which it exercises depends not only the rate of production obtainable from any construction outfit, but also the unit cost at which such production is secured. A man with real managerial ability, free to run the job as a whole, is essential to high rates of production, low unit costs, and best possible profits; but if the construction industry is to reap the full advantages which management offers in translating designs and plans into realities, it must also develop a spirit of mutually helpful co-operation and understanding between those who manage and those who plan and supervise. The relation of costs and quality are so interwoven that the ideal adjustment can never be attained except through mutual co-operation and assistance.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

CONSTRUCTION MANAGEMENT[1]

BY RICHARD HOPKINS,[2] ESQ.

SYNOPSIS

Management of construction work may well be classified into two departments: (1) The objective art of bidding for the contract, planning the work, and bringing it to completion; and (2) the subjective art of combining the elements in Part (1) to discover leaks in profit due to avoidable time losses in the various steps involved. This paper is confined to the first of these departments.

Standards of success in conservative bidding are discussed, as also are the ideal qualifications of the most important men on any job, namely, the superintendent, master mechanic, and the foreman. The paper deals, specifically, with the best attitude to hold concerning the maintenance of a construction schedule.

Intelligent bidding is the first requisite of good management. Anything that is well bought is half-way along the road to being well sold. A "lemon" on the day of the letting scarcely ever becomes a "ripe plum" when the final ledger accounts of the job are closed.

Each organization should appreciate what class of work it is best equipped to handle. Doing work in one locality only is generally not a good policy. Because two jobs lie close to each other is no reason why the same organization should attempt to build them, if they are dissimilar. Furthermore, in some years, the volume of work let may be small and the competition severe in the region where the contractor has been doing business. Prices, however, may be much better in another section. The contractor must meet the market price, but he can always choose the market in which to sell.

As a general rule, a man who obtains more than 20% of the work on which he bids is not a good bidder. He is bidding below the market. Good bidding is more often found among those who are awarded contracts for fewer than 10% of their bids. It is much better to let machinery depreciate, and parts of an organization stay idle, than it is to lose cash from a bank account. From two to six months before a job is planned to be finished the contractor should be looking over prospective work and putting in occasional

[1] Presented at the meeting of the Highway Division, New York, N. Y., January 16, 1930. Written discussion on this paper will be closed in December, 1931, *Proceedings*.

[2] Richard Hopkins Co., Albany, N. Y.

bids on desirable jobs. There is no more pitiful figure in the construction world than the man who has a smoothly running and efficient organization on a particular job and waits until a week or two before it is finished to start bidding on new work. The result is likely to be bad from three angles: The new work may not be the right kind, it may not be obtained in time, and the price may be much too low.

The head of an organization should be introspective and critical of the quantity of work that he can have under way at one time. Most contractors do too much work rather than too little. The financial strain and lack of proper attention both to general planning and small details are generally harmful to profits under such conditions.

It is assumed that the bidder makes up an estimated field cost which is based on his own records. It is further assumed that to this field cost he adds his estimated over-head which includes such items as interest, depreciation, insurance, repairs, workmen's compensation, camps, office, and freight and loading charges, in moving equipment to the job. This estimated total should be close to his real total cost if labor, the elements, and the supervising engineers all act in a perfectly natural way. At this point in making up the bid, the question of contingencies enters. What are the soil conditions? How much harm will a week of rain do to progress? Will some other contractor or some public service company be working on the job at the same time, and, if so, what are their records for co-operation? Are there bridges or other structures on the job that will slow down the entire program? The bidder should judge what these contingencies will amount to in dollars and add them to the previous total.

The bidder is then ready to decide what percentage of profit or what lump-sum profit he will add to his total estimated cost. The sum added for profit will depend on whether or not the bidder is "hungry." It will depend on whether or not the job is in a market that has more than its share of labor troubles. It may vary through a wide range, depending on whether it is largely a labor job or largely a material job, and on the definiteness or indefiniteness of the engineering inspection that can be expected.

Planning the Work

It is assumed that the contractor has a head office that directs all field operations on the different jobs and handles all administrative work. A superintendent may be assigned to a particular job because he is the only one available, or because the job is suited to him and his organization. If possible, the latter reason should be the one for giving him the job. The superintendent is the all-important man on the job. His personality is of vital importance, very much more so even than that of the contractor himself. It is not always possible to have a halo around the head of the superintendent, but all construction men are learning to appreciate that nothing pays better dividends than sentiment. The superintendent should be a real leader, a friendly czar, a man who knows all the letters of the construction alphabet. Above all, he should be a man who knows how to plan and how to get performance and who knows how to dovetail the different operations into a completed whole.

Every foreman should have such ability as to command the respect of his men. They may find fault with his temperament and with some of his methods, but they must respect his ability if he is to be successful. They must feel that he knows his job. There should be a foreman on each particular operation; there should be definite responsibility for each task. If only two men are working by themselves, one of them should have the responsibility for getting the task done in the right way. Co-operation among the foremen is a prime necessity. Personal feeling and jealousy between two foremen should never be allowed to interfere with the progress of the work.

A large number of contractors are coming to realize that it is wise to combine their engineering and accounting work on a job, and place both under the control of a trained engineer. There is a close inter-relation between accounting and engineering, and nobody should be able to interpret cost figures better than an engineer. His costs will be up to date, and, with his help, it will be possible to "lock the barn door before the horse is backed out of the stall." On account of his training, the engineer is able to suggest changes in methods and personnel and even in equipment when the proper costs are not being obtained. The items of extra work, special measurements, necessary departures from plans and specifications—all have engineering and accounting woven so closely together that it is best to have them handled by one man.

The selection, purchase, and care of equipment were never so important as at present. In 1910, or thereabouts, 85 to 90% of the cost of a grading job was the payroll. About 1915, when hard-surfaced roads came to be the National vogue, the payroll was still more than 50% of the total cost. Now (1931) all the major operations and most of the minor operations on a job are done by machinery. Steam machinery has been greatly improved, the automotive engine has taken a preponderant hold on all construction equipment, and the Diesel engine is making steady progress. No construction organization can long face the present competition without being machine-minded.

Next to the superintendent the master mechanic is the most important man on the job. He should be an all-round mechanic, of course, but with the present-day machinery he should be particularly competent on automotive work. He should have enough help to make quick repairs in the event of the inevitable break-down. Most of the minor repairs and some of the major ones should be made nights, Sundays, and rainy days. If the master mechanic is not given authority to employ and discharge machine operators, he should at least be consulted by the superintendent on both employment and discharge. When the purchase of any piece of equipment is contemplated, his opinion should be asked on three questions: (1) Is the machine staunchly built? (2) Is it well designed? (3) Is a large repair bill or a small repair bill being bought with the machine?

He should see and approve all repair bills before they are paid. He should have authority to keep on hand whatever repair parts his experience teaches him are most often needed. He should be "construction conscious"; and should appreciate thoroughly the earning power of each machine and the consequent loss when the machine is not in operation.

The importance of the master mechanic's duties has been particularly stressed, but no good master mechanic can show results unless he has the help of experienced and competent operators on the different machines. Good foremen can show only indifferent results when key machines have poor operators. The machine cannot function without a hand to guide it. Often the major portions of big jobs are run with old equipment that has long since been charged off, but in the hands of good operators it is earning dividends every working day. On the other hand, a visit to a similar job with new machinery, manned by incompetent operators, often reveals numerous breakdowns which cripple any effective schedule, and, at the finish of the job, there remains a pile of junk and a depreciation charge double or even triple what a reliable estimator had a right to expect. Physicians and hospitals exercise a big part in every-day life to keep the public in good health. The construction physician is the master mechanic and when he and his internes —the machine operators—all know their job, the piece of construction under their hands is in good financial health.

Saving one shovel, one roller, or any other piece of equipment by speeding up the production of those units working on the job, cuts down the machinery charge against a job. Loading a job up with unnecessary machinery increases the machinery charge. Moving, loading, and freight charges on heavy machinery constitute a cost that is not sufficiently appreciated. Frequently, if a heavy machine is to be transported a long way and the work required of it will take only a few weeks, it is cheaper to rent a machine close at hand.

A point about which there is a sharp difference of opinion among contractors is that of maintaining a schedule. All will agree that a job that drags is seldom profitable. One school of thought upholds the ideal of 110 cu. yd. of earth per hour for the small shovel and 50 cu. yd. of concrete per hour for the paver, and there is the continual drive to equal these figures on a few big days. A few trucks and a few teams as well as a number of men are added to the payroll. The other school of thought insists on an orderly progress of the work along the line of a well-planned schedule. There are no big days, but there are some good weeks and some wonderful months. All work proceeds at an orderly consistent pace. The advocates of this method argue that the job that is built the quickest is not always built the cheapest. Final unit cost is the yardstick, to decide on the relative merits of the arguments put forth by these two widely divergent points of view.

There is an old axiom that no well-conceived military operation can be a success without good infantry. No construction operation can hope to be successful without good laborers, who are satisfied with their tasks. Their length of service is measured in years—not in days or weeks—and turn-over is small. Their camps are clean, their food is good, they know that the superintendent is a fine boss and always treats them square. They boast a little about how much better they are than the outfit on the other side of the ridge. Underlying it all is the firm conviction that they know their job and that the boss knows his.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ENGINEERING FOUNDATION COMMITTEE ON ARCH DAM INVESTIGATION

ARCH DAM INVESTIGATION

Discussion

BY THE COMMITTEE, AND MESSRS. H. W. DENNIS, FRED A. NOETZLI, WILLIS A. SLATER, AND ALFRED D. FLINN

GENERAL STATEMENT FOR COMMITTEE[5] (by letter).[5a]—The Arch Dam Committee of Engineering Foundation acknowledges gratefully the contributions to the development of its subject made so generously by those who have formally discussed Volume 1 of its report, published as Part 3 of the *Proceedings* of the Society for May, 1928, and by those who have added to knowledge of arch dams and of the use of models by studies, experiments, and experience reported in separate papers or through channels other than the Society. It is noteworthy that some of the discussions have come from men of other countries, just as did contributions of information and suggestions in the earlier stages of the Committee's enterprise. It is particularly encouraging to note that the Committee's endeavors to aid in solving some of the problems of design and construction of arch dams have inspired other engineers in several parts of the United States, and in other lands, to undertake experimental observations on dams being constructed or on dams previously in existence. One or two are known, also, to have undertaken experiments with models.

Mention must be made, too, of the continuing co-operation of certain bodies that have generously supplemented the Committee's work from an early date, namely, United States Bureau of Reclamation, National Bureau of

NOTE.—The Report of the Committee of Engineering Foundation on Arch Dam Investigation was published in May, 1928, *Proceedings*, Pt. 3. Discussions of the Report have appeared in *Proceedings*, as follows: November, 1928, by Messrs. Roy M. Green, Harry O. Wood, and R. A. Sutherland; January, 1929, by Messrs. Camillo Guidi, A. Nadai, Nils Buer, and William M. Kinney; April, 1929, by Messrs. B. A. Smith, E. Probst, and Lars R. Jorgensen; May, 1929, by B. F. Jakobsen, M. Am. Soc. C. E.; August, 1929, by Messrs. Roy W. Carlson, and J. L. Savage and Ivan E. Houk; May, 1930, by Fredrik Vogt, Assoc. M. Am. Soc. C. E.; and March, 1931, by Eugene Kalman, M. Am. Soc. C. E.

[5] For names of Committee see *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 1.

[5a] Received by the Secretary April 27, 1931.

Standards, University of California, University of Colorado, and the Southern California Edison Company, Ltd. The results of these supplemental efforts and of the recent work of the Committee will be presented in the second volume of its report and should satisfy in part desires for further information expressed in the discussion of Volume 1. Detailed responses to the discussion have been written by the authors of the different chapters in Volume 1. All these parts of this closure have been assembled and reviewed by the officers of the Committee.

Four general additions to the art of dam building appear to have resulted from the Committee's work hitherto:

1.—Certain facts that have been experimentally determined.
2.—Improvement of technic for experimental investigation of dams, including instruments.
3.—Demonstration of the usefulness, economy, and dependability of small models made and tested in accordance with the laws of similitude.
4.—Development of materials having suitable qualities for models of various scales and conditions of testing; also instruments and technic for testing models. (Some of this information will appear in the next volume of the Committee's report.)

To which may be added, a new impetus to the experimental and mathematical study of dams of all kinds.

Instrumental problems have not all been solved. Especially there is greatly desired an instrument, or a combination of instruments, of permanent reliability which can be embedded at desired places in concrete or other masonry dams and read at stations outside the dams and possibly somewhat remote therefrom, which will give, upon observation, or will record continuously, temperatures and strains at the places of embedment. Instruments are desired also that can be attached to the up-stream face of a dam, which will indicate or record (at a convenient place of observation) strains, temperatures, and other information whenever desired over a period of years and which will not be affected by exposure to weather or continuous submergence to depths ranging from a few feet to several hundred feet. The Committee invites suggestions from instrument-makers and others, and will give particulars to persons interested in working on these problems.

Mr. Jakobsen expresses the hope that a new large-scale test can be undertaken. A few others have conveyed to the Committee a similar desire to increase the height of Stevenson Creek Dam and to test it to destruction. In either form the suggestion involves a large expenditure for which Engineering Foundation has no funds. If the funds can be provided and the further direct information secured, then the additional comparison with small-scale models might be worth while, in view of the variety of forms and conditions for arch dams and the great number and importance of arch dams being built and likely to be built in years to come. However, there have come to the Committee an increasing number of expressions of confidence in results obtained with small-scale models and of belief that much more knowledge could be gained from equal or less expenditure for tests of small models.

Mr. Sutherland proposes a testing laboratory for models. Similar proposals were made by others a few years ago and received study by Engineering Foundation, but the time was not then ripe. Recent progress in the art now offers greater prospect of success to encourage some laboratory to undertake this service. The service should be self-supporting, through suitable charges. One of the engineering colleges would appear to be a natural place. There would be obvious advantages in concentrating this work in one or two institutions, according to the volume which might develop and necessary convenience of accessibility. The Committee could be helpful in an advisory way in determining the suitability of laboratories that might offer themselves or that might be proposed, and in indicating the nature of the service, the equipment required, and desirable qualifications of personnel.

Such a model-testing laboratory would not be limited to arch dams, but could deal with many other types. Besides investigations of proposed dams as a method of determining designs, or of checking designs based on computations, the margins of stability of existing dams, new and old, could be tested in models to supplement studies of other features of safety covered by investigations conducted on behalf of owners or for public supervisory bodies. Ordinarily, the cost of the model-testing service would be trifling when compared with the cost of the dam or the value of the lives and property involved in considerations of the security of any dam. Furthermore, through the work of such a laboratory substantial contributions should be made to the science and art of dam building, both in economy and in security of the dams. Hydraulic testing of models of dams has been carried on in recent years at Worcester Polytechnic Institute, Massachusetts Institute of Technology, and other institutions, with great benefit. This experience in a related field should lead owners, promoters, and public authorities to be receptive toward a service for utilizing models to add to the assurance of the adequacy and economy of the structural aspects of a proposed or existing dam.

Dr. Vogt interestingly calls attention to the great increase in the use of single-arch dams since 1910, and the almost total freedom from failures of dams of this type through a history of several centuries. Indeed, if the Committee's information is complete and reliable, the very few reported instances were not failures of the dams, but of the foundations, or the natural abutments, or the earth embankments built in connection with the arch dam.

Through this discussion and other channels, suggestions have repeatedly been made for thorough, scientific investigations of gravity and earth dams. The latter subject might appropriately be included in the program of the Society's Special Committee on Earths and Foundations. For the study of gravity dams, the Society might well constitute a competent committee and call upon Engineering Foundation for assistance in planning, organizing, and financing a co-operative research project.

The need for construction of dams of a number of kinds will continue for an indefinite future, in many lands. Always a very important desideratum will be assurance to those affected by the dam in any way that it is safe and will remain safe through a very long period. Such assurance may

have to be repeated as generations pass. Results of investigations of the sort being conducted by the Committee are among the most important means for increasing and for demonstrating the security of dams and, therefore, for persuading the public to continue to permit the building of dams.

Although this work has now continued through a number of years, the Committee does not yet feel warranted in stating definite conclusions. Many of its observations are not yet completed, nor the results fully digested. The portions of its report yet to be published must be exposed to discussion for the benefit thus to be reaped. Owing to the impracticability of complete experimental or mathematical solution of a problem having so great a multiplicity of phases, there is room for variety of opinion. There has been such variety in this discussion and in other writings on the Committee's work, and it has been helpful. It is to be hoped that within a relatively near future additional data and discussion will enable the Committee to state some deductions and even conclusions in a form convenient for use by designers and other engineers which will merit reasonably general acceptance and which may become the basis for further progress. The Committee's primary purposes have been to ascertain, collect, and state facts, to make information available, and to stimulate study of dams by competent engineers and scientists.

H. W. DENNIS,[c] M. AM. SOC. C. E. (by letter).[cc]—In the report of the Committee, Paragraph 46 of Part IV gives a description of the condition of the dam as of July, 1927, and Fig. 104 shows the important cracks known to exist in the dam as of that date. There was no significant change as of April 1, 1931, 36 ft. north of the center line, in the cracks as shown in Fig. 104, but there was an additional vertical crack which was not shown in that diagram. It extended across the crest of the dam and downward at least 5 ft. on the up-stream side. The water stood at an elevation 5 ft. below the crest, and there was no facility to permit any determination as to the actual total length of the crack below the water surface. Its measured width was about 0.01 in., but it could not be seen at all on the down-stream face. It should be borne in mind that there is no facility for a careful examination of the down-stream face except by the use of field glasses, but if the facilities were available this crack might appear. On June 21, 1931, there was no water standing behind the dam and the writer was able to trace this crack on the up-stream face down to its apparent terminus at Elevation 48 (12 ft. below the crest). It was found to be vertical all the way.

The writer has examined the dam at various times since July, 1927, and this crack was first noted by him in September, 1929. At that time, and at subsequent times, he has examined the corresponding point on the other side of the center line and has failed to find any evidence of a crack in that location. Fig. 104 indicates an unsymmetrical disposition of the débris at the up-stream face of the dam. This lack of symmetry still obtains, and it may be significant that the crack in question occurs where the surface of

[c] Chf. Civ. Engr., Southern California Edison Co., Los Angeles, Calif.
[cc] Received by the Secretary April 27, 1931.

the débris is from 15 to 20 ft. below the crest, whereas at the corresponding point south of the center line, the débris is less than 10 ft. below the crest. The crack being open on the up-stream side indicates up-stream movement of the crest of the dam, and the absence of a crack at the corresponding point may indicate that the débris actually restrains the dam against an up-stream movement at that point.

On April 1, the vertical crack extending downward from the crest at the center line was open from 2 to 3 mm. at the down-stream face, and 0.5 mm. at the up-stream face, where the crack showed down to and below the surface of the water. On the down-stream side the crack shows plainly down to Elevation 45, and may possibly extend lower. Fig. 104 indicates furthermore that at the time of the previous examination this crack extended down to Elevation 40. The vertical crack on the center line of the dam, extending up from the foundation, is traceable from the foundation up to Elevation 18.5; and, similarly, the crack extending diagonally up from the foundation north of the center line may be traced up to Elevation 18.5. This seems to be a little higher than is shown in Fig. 104 as of July, 1927, although the crack which extends upward diagonally from the foundation south of the center line may be traced to Elevation 20.5, which seems to agree with the condition of that crack as shown in Fig. 104.

The only other differences are the horizontal cracks, which in general follow the days' work joints. Fig. 187 shows such of these cracks as were readily discernible, and by shaded areas it shows those parts of the down-stream faces of the dam that are wet by reason of leakage through these cracks.

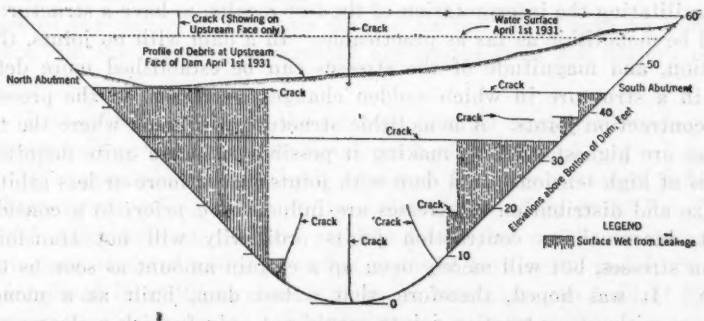

FIG. 187.—CONDITION OF STEVENSON CREEK DAM, APRIL 1, 1931.

It was observed that the cracks at the foundation line on either abutment show from the crest down to an elevation below the water line where actual leakage occurs. This leakage follows the contact between the concrete of the dam and the foundation and obscures the opportunity for observation of this crack at the lower half of the dam.

Since the storm in November, 1926,[7] there has been no means of controlling the water entering the pool behind the dam. A part or all of the water has

[7] Report of the Committee of Engineering Foundation on Arch Dam Investigation, Vol 1, *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, Paragraph 46, Pt. IV, p. 142.

found its way through the débris deposit and has passed out through the under-sluice. At times the flow has been sufficient to overtop the dam, but generally the elevation of the water in the pool behind the dam has been at about 5 ft. below its crest.

FRED A. NOETZLI,[8] M. AM. SOC. C. E. (by letter).[8a]—In the discussion of the results of the tests on Stevenson Creek Dam some questions were raised regarding the design and the layout. It seems in order, therefore, to emphasize once more the fact that the dam was planned primarily for testing purposes, and, consequently, departure from certain principles of well-established practice was unavoidable. Among these should be mentioned the extremely small thickness of the dam, the relatively small central angles of the present 60-ft. structure, and the omission of vertical contraction joints.

The thickness was determined on the basis of the requirement that, at an initial height of 60 ft., the dam should be relatively "safe" for testing purposes, while subsequently the structure should be reasonably certain to break under load if and when raised to a height of 100 ft.

As a natural consequence of the narrow V-shaped canyon on Stevenson Creek, the central angles of the arches of the present 60-ft. dam are relatively small. Above the 60-ft. elevation they would have become increasingly larger until at the 100-ft. height even a semi-circular arch would not have reached completely across the canyon. As indicated on the plan in Fig. 1, a portion of the right-hand abutment of the 100-ft. dam, therefore, was planned to be supported by a gravity tangent.

There were several definite reasons for not providing any vertical contraction joints. First, it was most desirable from the point of view of testing and facilitating the interpretation of the test results, to have a structure which would be monolithic as far as practicable. In a dam with no joints, the size, direction, and magnitude of the stresses can be established more definitely than in a structure in which sudden changes occur due to the presence of open contraction joints. A monolithic structure will crack where the tension stresses are highest, thereby making it possible to locate quite definitely the regions of high tension. In a dam with joints placed more or less arbitrarily, the size and distribution of stresses are influenced *a priori* to a considerable extent, inasmuch as contraction joints ordinarily will not transmit any tension stresses, but will merely open up a certain amount as soon as tension occurs. It was hoped, therefore, that a test dam, built as a monolithic structure without contraction joints, would not only furnish a clearer picture of the size and distribution of stresses, but also at the same time would permit conclusions to be drawn on the proper location of contraction joints in future dams to be built for service. In both respects the tests have furnished quite satisfactory results.

After the dimensions of the test dam had been determined by trial computations to satisfy the requirement of probable failure under load of the 100-ft. structure, the design stresses were analyzed in two ways: First, by

[8] Cons. Hydr. Engr., Los Angeles, Calif.
[8a] Received by the Secretary April 27, 1931.

the ordinary "cylinder formula," considering the horizontal arch elements as structurally independent segments of a thin cylinder; and, second, by a tentative method based on the assumption of a series of horizontal arch elements and one vertical cantilever slice at the center of the dam. In this connection a brief review of the history of the test dam project may be of interest.

Previous to about 1920 most arch dams were designed by the simple "cylinder formula." Efforts by various writers to introduce more scientific methods of design had produced little practical result, possibly due to the fact that the theories were somewhat involved mathematically. Certain other methods, when applied to several arch dams in service, indicated the probability of high-tension stresses and possibly cracks existing in these dams, especially at the base of the vertical cantilevers. The publication of these deductions gave rise to rather violent expressions of disagreement in certain quarters, which prompted the writer to appeal to Engineering Foundation for a thorough research by gathering data from existing arch dams and by building a special experimental dam to be tested to destruction. The favorable support of this proposal by Director Flinn and many other engineers in all parts of the United States resulted in the appointment of the Committee on Arch Dam Investigation by the Foundation Board.

In 1923, the late W. A. Brackenridge, M. Am. Soc. C. E., Senior Vice-President of the Southern California Edison Company, proposed the construction of a test arch dam on Stevenson Creek, and in behalf of that Company, offered a large contribution of funds and other facilities for building such a dam. It fell to the writer as Secretary of the Arch Dam Committee to prepare the design of the dam.

Some of the results of the tests bearing on the design may be worthy of emphasis, and the following paragraphs are quoted from Mr. Slater's part of the report:

1.—"It is quite obvious that the cylinder formula is entirely inadequate to represent conditions found in an elastic arch of this kind." [9]

2.—"The agreement of the loads obtained from the observed deflections with those corresponding to the design deflections is fairly good." [10]

3.—"A crack between the dam and the foundation on the up-stream face at the bottom occurred under a head of water of between 30 and 40 ft. This crack increased in width to about 0.05 in. at the maximum head of 60 ft." [11]

The experiments further established the fact that the water load upon the arches is not uniform between the crown and the abutments. In the V-shaped site on Stevenson Creek the arches at the crown deflected more than was indicated by theory for an arch loaded with uniform normal load. The interesting tests made in Italy by Professor Guidi on a model of an arch dam between vertical side abutments, on the other hand, showed deflections which at the crown were considerably less than those computed for uniform

[9] Report on Arch Dam Investigation, Vol. 1, *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 189.

[10] *Loc. cit.*, p. 191.

[11] *Loc. cit.*, p. 192.

normal load. It is probable, therefore, that at certain sites of a shape intermediate between a V and a U the actual and the theoretical deflections will be in closer agreement with each other than on either Professor Guidi's model or on the Stevenson Creek Dam.

Mr. Sutherland also calls attention to the influence of the shapes of different dam sites upon the division of the water load between horizontal and vertical elements, and suggests that the load distribution might be expressed approximately in terms of the shape of the dam. If a simple solution should be feasible (which, however, is doubtful), it might be used to advantage for the purpose of arriving at a first approximation, for instance, for use with the trial-load method.

Professor Probst points out the feasibility of certain modifications of arch dam design and layout so as to avoid some of the difficulties encountered with arch dams of the ordinary type. His proposal is to build arch dams as a combination of a gravity base, a very plastic intermediate strip, and an upper arch free to move relatively to the gravity base. This type of construction would tend greatly to reduce the restraint of the cantilevers upon the arches and, therefore, it would be beneficial in several respects.

Mr. Jakobsen presents some interesting computations relative to the stresses in the arch at Elevation 30. By the method of least squares he attempts to compute the most probable values of the arch loading and of the shrinkage of the concrete. By this process he arrives at a value of an equivalent temperature drop of $t_0 = -47.3°$ Fahr.[12]

In this connection it is of interest to note that during the tests in question, which took place between about midnight and 8:00 A. M., the temperature of the dam varied generally less than 1° cent.[13] Furthermore, owing to the installation of a sprinkling system along the crest of the dam, which kept the concrete wet day and night practically throughout the entire testing period, shrinkage of the concrete was probably largely, if not entirely, offset by the effect of continuous water-soaking. Mr. Jakobsen's comparison between the design stresses and his results, including his computed shrinkage stresses, is, therefore, evidently based on erroneous assumptions.

Mr. Jakobsen apparently misunderstood the load diagram shown in Fig. 4. The area corresponding to the assumed drop of temperature of — 20° Fahr., is properly marked on this diagram. The writer regrets not having explained more fully that this "load" is not an actual one, but merely a load equivalent to one producing the same arch deflections. In the subsequent analysis, "the transfer of load from the arches to the cantilevers, or *vice versa*,"[14] was properly considered.

Professor Kalman's discussion presents many points of interest. Most of the questions brought up have been answered by Professor Slater. Professor Kalman suggests[15] that "it should be possible to design the bottom

[12] *Proceedings* Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1244.

[13] Report on Arch Dam Investigation, Vol. 1, *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 124.

[14] *Loc. cit.*, p. 1246.

[15] *Proceedings*, Am. Soc. C. E., March, 1931, Papers and Discussion, p. 435.

arches with a reduced section that permits them to deform under load as the need arises, without a break." He further inquires,[16] "may not a structural element fail because it has been made so thick that it is prevented from transmitting part of its own weight by means of its flexibility to adjacent structural elements?"

The writer is in full agreement with Professor Kalman on these questions. As a matter of fact, the designers of several arch dams built during the past few years have already made practical application of these principles, partly guided no doubt, at least in some cases, by the results of the test dam investigation. Among the arch dams built with reduced cross-sections, and characterized by a slight overhang down stream, are the Calles Dam, in Mexico; the Stewart Mountain Dam, in Arizona; and the Railroad Canyon, Calaveras, and Hansen Dams, in California.

In the discussion by Messrs. J. L. Savage and Ivan E. Houk an interesting comparison is given of an analysis by the trial-load method with results from the tests. Both for load distribution and for stresses in arches and cantilever elements, the agreement is remarkably close.

Similar results were attained by Mr. Carlson by a somewhat different method. He made two analyses: First, by assuming arches and cantilevers as "partly fixed" at the abutments and the base, using the closing of cracks and yielding of the abutments as measured on Stevenson Creek Dam as criteria; and, second, by assuming arches and cantilevers as "fixed" and with supports unyielding. An interesting comparison between test data and theoretical analysis was further made by Mr. Smith.

These several contributions to the discussion in connection with the test results on the dam itself disclose a most remarkable situation:

First.—Tests on models and theoretical analyses, whether based upon the assumption of complete "fixity" of the dam at the base and at the abutments (Professor Beggs' celluloid model, Mr. Carlson's second analysis, and Mr. Smith's method), or upon the assumption of cracks in the dam and elastical deformations of the bed-rock (trial-load method and Mr. Carlson's first analysis), give results which agree surprisingly closely with each other and with the data from the tests on the dam.

Second.—High-tension stresses and even cracks, both in the arches and cantilevers, seem to be unavoidable in the usual type of arch dams, except perhaps at especially favorable sites.

Third.—Such tension stresses or cracks apparently do not weaken the structure to a dangerous degree as long as the section of the concrete that remains in compression is capable of carrying the load with adequate safety against crushing.

Final conclusions may not yet be warranted, but in the writer's opinion one of the outstanding results of the arch dam investigation thus far is the fact that these important deductions have been established beyond reasonable doubt.

[16] *Loc. cit.*, p. 435.

WILLIS A. SLATER,[17] M. AM. SOC. C. E. (by letter).[17a]—The interesting discussion of "Part IV.—Report of Tests on Stevenson Creek Dam," has been a source of satisfaction to the writer. Unfortunately, it has not been possible to go deeply into some of the analytical phases presented by various discussers. This closing discussion is confined largely to matters of fact regarding the test data and to the more apparent applications of them in an analytical form.

The assumption by Mr. Smith, that a drop in temperature of 10° cent., giving an assumed tension of "54 lb. per sq. in., which may account for a crack that developed" on the vertical center line, at the top of the dam,[18] seems to be unwarranted for several reasons. Of course, it is possible that shrinkage and temperature stresses, added to those caused by the applied load, may have hastened the formation of the crack; but the fact that, when the reservoir was drained, the crack nearly closed so that it could scarcely be discovered, indicates that the temperature stress was not the controlling element.

A 12 by 12-in. beam made of concrete similar to that used in the dam, and cured in the same manner, was stressed to 216 lb. per sq. in. in tension without failure.[19] The tensile stress in the upper part of the dam just prior to cracking, was about 240 lb. per sq. in., as given by telemeter readings, and that in the lower part just prior to cracking, was 520 lb. per sq. in.[20] From these facts it appears that a tensile stress of 54 lb. per sq. in. could not have been the principal cause of the crack in the upper part of the dam.

Mr. Smith makes an analysis of a cylinder under external pressure, assuming that the generators of the cylinder remained straight and that there was no extension of the horizontal elements. Based upon these assumptions, he has computed deflections which are in reasonable agreement with those which were found in the test. This is of interest, but in view of the fact that the measured deflections showed that the generators of the dam did not remain straight, it is not evident that the application of this analysis to the Stevenson Creek Arch Dam is justified. Furthermore, it is not evident under these conditions that the "bending moment, M, is zero throughout," nor is it evident that the "arch thrust, T, is also zero," when there was a measured compressive deformation in the abutment rock which amounted to nearly 0.04 in. (see Fig. 82). If this compression of the rock were due to the bending stresses in the arch rings, there should be a tension on the up-stream side of the arch at the abutment equal to the compression on the down-stream side at the same place. As there was no appreciable tensile resistance between the arch and the abutment, it is not apparent how such a condition could exist.

The relative movements of the concrete and the abutment rocks on the up-stream face were not measured during the load tests, due to the presence of the water, but the fact that from Elevation 30 to Elevation 36, and from

[17] Research Prof., Eng. Materials, and Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[17a] Received by the Secretary April 10, 1931.

[18] *Proceedings*, Am. Soc. C. E., April, 1929, Papers and Discussions, p. 1008.

[19] *Loc. cit.*, May, 1928, Pt. 3, p. 154.

[20] *Loc. cit.*, p. 121.

Elevations 53 to 56, where the shrinkage crack "was covered with fresh mortar, the mortar did not crack,"[21] indicated that there was no appreciable tendency to crack. The writer has the utmost respect for the value of purely analytical treatment of the structure, but it seems that the physical evidence of bending moment and thrust in the Stevenson Creek Dam is too great to admit of the application to this structure of the results of an analysis which has as a fundamental assumption that there was no bending moment in the vertical elements and, as a resulting conclusion, that there was no thrust.

In discussing the up-stream deflection in certain parts of the upper arch elements Mr. Jorgensen stated,[22] that "the action is like that of a long column with failure impending." That this up-stream deflection was not a buckling due to the compressive stresses is indicated by the fact that a crack across the entire thickness of the dam occurred at the crown in this same arch element. By combining the total horizontal strain at the 60-ft. elevation from Fig. 94 with the bending strain at the same place from Fig. 124, it appears that the direct compression in the arch element ranged from about 35 to 45 lb. per sq. in. Even with a head of 30 ft. of water there was some up-stream deflection at the 60-ft. elevation and, at the same time, the horizontal strains at this elevation were too small for accurate measurement. The up-stream deflection seems, therefore, to have been purely a flexural, and not a buckling, phenomenon.

Mr. Jakobsen has examined searchingly, by means of the Cain formulas,[23] the conditions in the arch ring at the 30-ft. elevation. Fig. 136 indicates that the measured horizontal strains at this elevation lay between those computed by the Cain formula for an arch with fixed ends and those for free ends, and to this extent the writer agrees with Mr. Jakobsen's conclusion[24] that the results of the test furnish "a verification of the Cain formulas." He is in complete agreement with Mr. Jakobsen's statement[25] that "a serious discrepancy between theoretical and test values * * * could not be accepted as proof that the theory is incorrect," because there are so many chances for conditions of the tests to be out of harmony with the assumptions which underlie the theory.

Mr. Jakobsen's computed stresses apparently include combined shrinkage and temperature stress equivalent to the effect of a drop in temperature of 47° Fahr. There is no doubt that shrinkage and temperature stresses existed as is indicated in Figs. 46 to 64, and it is probable that the behavior of the dam was affected thereby. However, these temperature and shrinkage stresses were present before the no-load readings were taken. Only the temperature change and shrinkage occurring between the "no-load" readings and the "load" readings would affect the strains measured in the test and these temperature changes did not generally exceed 1° Fahr. (see Fig. 47). Since water was kept running over the surface of the dam previous to filling the

[21] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 120.
[22] *Loc. cit.*, April, 1929, Papers and Discussions, p. 1019.
[23] "The Circular Arch Under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.
[24] *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1244.
[25] *Loc. cit.*, p. 1241.

reservoir, there could have been little shrinkage or expansion. It is true, of course, that the stresses caused by the water pressure must have been different from what they would have been if the cracking away from the abutments had not occurred due to the early temperature and shrinkage deformations indicated in Figs. 46 to 64. This effect, however, is due to a change in the character of the structure under test and cannot be corrected by any assumed temperature change.

It was the writer's function in the test to evaluate the effect of the temperature and shrinkage strains separately from the stresses, due to the water pressure. The former was done as far as practicable in Sections 32 to 37 and the latter in Sections 38 to 70 of Part IV of the report. It is Mr. Jakobsen's function as a designer, to determine the total effect of the strains due to temperature change and shrinkage and those due to load. For comparison with the results of the load tests, however, the effect of the load should have been determined separately from that of temperature and shrinkage and then the effect of temperature and shrinkage should have been added. It is surprising, therefore, to find from his analysis that the total effect of the water pressure plus a drop in temperature of 47° Fahr., on the horizontal stress in the down-stream face agreed so closely with the observed stresses due to the water pressure alone. The two sets of stresses are given in his Table 50. Either important conditions qualifying the applicability of the Cain formula have escaped Mr. Jakobsen's attention, or the stresses due to temperature changes are much smaller than the writer supposed.

The writer agrees with Mr. Jakobsen's comment[26] that if, as is indicated by the tests made by Professor Davis, "the plastic flow of concrete increases in direct proportion with the stress, * * * the plastic flow will have little or no influence on the stress distribution." Mr. W. H. Glanville has confirmed the finding of Professor Davis as to proportionality between stress and plastic flow.[27] However, since the shrinkage and temperature deformations are independent of the load, their effect must be evaluated separately from that of water pressure.

The discussion[28] presented by Messrs. Savage and Houk shows that the trial-load method developed by the U. S. Bureau of Reclamation, when applied to the test dam, gave deflections and strains, both of which agreed very well with the deflections and strains measured in the test dam. The only important divergence of the observed from the computed deflections is in the upper central portion of the dam. It seems reasonable to attribute a large part of this divergence to the presence of the crack on the vertical center line in the upper 12 to 15 ft. This conclusion is strengthened by the fact that the deflections in these regions, as computed by Messrs. Savage and Houk, agree much more closely with the deflections calculated by Professor Beggs in his Table 31,[29] from the results of the tests of the celluloid model, than they do with the deflections of the Stevenson Creek Dam. The points

[26] *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1250.
[27] "Creep or Flow of Concrete Under Load," Dept. of Scientific and Industrial Research. London, England; *Technical Paper No. 12* on "Building Research."
[28] *Proceedings*, Am. Soc. C. E., August, 1929, Papers and Discussions, p. 1579.
[29] *Loc. cit.*, May, 1928, Pt. 3, p. 223.

of agreement and divergence are shown in Fig. 188, in which have been assembled the deflections computed by Messrs. Savage and Houk, those found in the tests of the Stevenson Creek Dam, and the celluloid model, as well as those reported by Messrs. Smith and Carlson. These comparisons tend to give confidence in the trial-load method of design, as presented by Messrs. Savage and Houk.

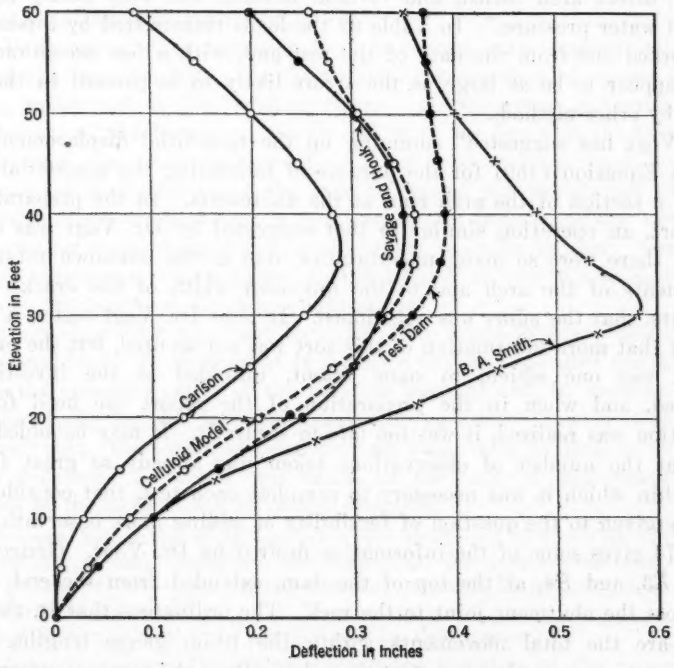

FIG. 188.—COMPARISON OF DEFLECTION TESTS.

The principle underlying the trial-load method used by Mr. Carlson appears to be the same as that used by Messrs. Savage and Houk, and Mr. Carlson's failure to secure as good agreement with the test data as did Messrs. Savage and Houk, probably is due to the fact that he applied the condition of equal deflections in arch rings and the cantilevers for only the central vertical element, whereas Messrs. Savage and Houk apparently imposed this condition upon several vertical elements, although they do not state how many were used for this purpose.

It seems that the method used by Messrs. Savage and Houk is exactly the reverse of that used by the writer in analyzing the results of the test. They determine what loads on vertical and horizontal elements would give equal deflections at the intersections of elements. The writer started with the measured deflections and, by a reverse process, determined the load distribution which caused these deflections. In both cases the influence of the torsion was neglected and the modulus of elasticity of the concrete was

taken as 3 600 000 lb. per sq. in. Therefore, the results should be in agreement whenever the imposed conditions of equal deflections at intersections of vertical and horizontal elements have been met.

The writer agrees with the conclusion by Messrs. Savage and Houk,[30] that the "transmission of load by tangential shear and twist could not have been important," since "in all cases the sum of the loads carried by horizontal bending, direct arch thrust, and vertical bending was very nearly equal to the total water pressure." In Table 22 the loads transmitted by torsion have been worked out from the data of the test and, with a few exceptions, they do not appear to be as large as the errors likely to be present in the loads derived by other methods.

Dr. Vogt has suggested[31] summing up the tangential displacement, Δt, given in Equation (150) for the purpose of estimating the tangential movement of a section of the arch ring at the abutments. In the preparation of the report, an operation similar to that suggested by Dr. Vogt was carried out, but there were so many uncertainties, due to the unknown rotation of the elements of the arch and to the unknown width of the cracks at the abutments, that the effort was abandoned. It is as Dr. Vogt states, a source of regret that more information of this sort was not secured, but the program followed was one which, to some extent, unfolded as the investigation progressed, and when in the preparation of the report the need for this information was realized, it was too late to secure it. It may be added, however, that the number of observations taken was already so great for the time within which it was necessary to complete each test, that consideration had to be given to the question of feasibility of adding more observations.

Fig. 74 gives some of the information desired by Dr. Vogt. Gauge lines, *E*1, *E*2, *E*3, and *E*4, at the top of the dam, extended from the end of the dam across the abutment joint to the rock. The ordinates—that is, the total strains—are the total movements within the 10-in. gauge lengths. The measurements apparently were started on June 13 at the south abutment, and on June 24 at the north abutment. The latter was the date on which the sprinkling water was temporarily shut off, and on which the downward movement began. All these gauge lines show that the cracks must have been partly open at the time the measurements were taken. Since the stresses in the concrete and in the rock at this point were exceedingly small, the negative changes in length must have been principally due to the opening of joints. Using the lowest point on each of these gauge lines as the best available datum plane, the indication is that, on September 20, the abutment joint at the top of the dam at the south end was open approximately 0.035 in. and that, at the north end, the opening was approximately 0.015 in. This is in reasonable agreement with Dr. Vogt's average value[32] of 0.025 as estimated from Equation (150).

[30] *Proceedings*, Am. Soc. C. E., August, 1929, Papers and Discussions, p. 1580.
[31] *Loc. cit.*, May, 1930, Papers and Discussions, p. 984.
[32] *Loc. cit.*, p. 983.

Dr. Vogt suggests[33] that the cracks in the upper part of the dam have caused "the greatest part of the difference at the crest between the measured deflection and the deflection computed in the design of the dam and shown in Fig. 126." That this is partly true is indicated by the fact that the deflections based upon the test of the celluloid model were smaller in the region of the upper center line than were those of the test dam (see Fig. 187), and by the fact that the deflections computed by Messrs. Savage and Houk agree well with those for the celluloid model.

Dr. Vogt states[34] that Equation (28) is correct only if there is no horizontal strain from bending. To the writer, it seems that he means that the term, ϵ_h, must include the bending strain. As there was a vertical crack at the bottom of the dam when the strains, with the water at the 60-ft. level, were observed, the horizontal strains due to bending must have been very small and the applicability of the formula would not be affected in either case. This statement, however, is not intended as defense of the correctness of the bending moment on the vertical element near the bottom. The results for these regions are characterized in the report[35] as very doubtful, but they seem to be the best that the data would yield.

It is not clear why Dr. Vogt believed[36] that Table 22 gives only one-half the torsional load. In Section 51 it is shown that the so-called torsional load is twice the cross-derivative of the twisting moment and, in working out the values for Table 22, this fact was taken into account.[37] Nor is it clear why he states[38] that "the formulas developed are only valid as long as the thickness is constant." Account was taken of the lack of uniformity of the thickness of the dam in the following sentence,[38]

"However, below the 30-ft. elevation the thickness of the dam varies with the elevation, y, and, therefore, the coefficient, EI, is not constant. Hence, in this portion of the dam, the differentiations must be performed alternately with respect to x and y."

The differentiations were carried out in this manner.

In numerous places Dr. Kalman questions the time of the occurrence of the vertical crack at the top of the dam, and he quotes[39] the writer as stating "that the crack could not have occurred under a head of less than 50 ft." The writer had no convictions and did no theorizing as to when the crack could have occurred, but stated his best information as to when it did occur. The behavior of the telemeter at Station 29 at the 60-ft. elevation, indicated clearly[40] that no crack was present at a head of 47 ft. Fortunately, this reading was recorded, but certain other readings were not recorded because these observations were taken rapidly on a large number of telemeter stations for the purpose of catching promptly the first sign of a crack or of other unusual behavior. It is because these intermediate readings were not recorded

[33] *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 984.
[34] *Loc. cit.*, p. 985.
[35] *Loc. cit.*, May, 1928, Pt. 3, p. 179.
[36] *Loc. cit.*, May, 1930, Papers and Discussions, p. 986.
[37] The calculations for these tables are on file in Engineering Societies Library.
[38] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 151.
[39] *Loc. cit.*, March, 1931, Papers and Discussions, p. 445.
[40] *Loc. cit.*, May, 1928, Pt. 3, p. 121.

that it is not known for certain whether readings on this station were taken at heads between 47 and 50 ft. With the water 10 ft. or more below the top of the dam there was nothing to attract particular attention to this station so near the top, as will be apparent from the fact that, with the head of 47 ft., the observed stress at the 60-ft. elevation was only about 240 lb. per sq. in., whereas a tensile stress of 520 lb. per sq. in. was observed before the vertical crack near the bottom occurred. When the head reached 50 ft. the discharge valve was opened slightly to hold the level constant and considerable vibration resulted. It is known definitely that the crack was present when the head was 50 ft. and the conditions described warrant the statement[41] that "it does not seem likely that it [the crack] occurred before the water reached a head of 50 ft."

Dr. Kalman states[42] that the crack "seems to have been caused by the shear." While the ability of the concrete to resist shear had a part in the formation of the crack, his statement seems misleading. Obviously, the crack was a tension crack caused largely by the bending moment. It was caused by shear only in the sense that shear in the horizontal planes transmitted bending load to elevations above the level of the water. The writer understands that the expression, "neglect of shear," as ordinarily used, refers either to the neglect of any danger from the shearing stresses as such, or to the neglect of the shearing deflection when using the total deflection for the estimation of bending moments. From Figs. 126 to 129 it may be found that the maximum shearing stress on the cantilevers was at no place greater than about 35 lb. per sq. in. Shearing stress as such would not be a serious danger until it reached an intensity about equal to the tensile strength of the concrete. Section 50 gives the writer's reasons for believing that the effect on stress distribution of neglecting the deflection due to shear was unimportant. Nothing in the test results changes his conclusion on this point.

In speculating on the formation of the vertical crack near the bottom of the dam, Dr. Kalman questions[43] "why the maximum tension should have been found to occur at the center instead of at the ends." As cracking away from the abutments was quite general during the curing period (see Section 36), it is not unlikely that at the 5-ft. elevation the dam was separated from the abutments sufficiently to permit action as a simple beam. Loaded uniformly to give a down-stream strain at the center of the span of 144 millionths $\left(\frac{520}{3\,600\,000}\right)$[44] a simple beam would have a total computed extension of the down-stream face within one-half the 30-ft. length of the arch at this elevation of 0.017 in. This is the amount of separation from the abutment that would be necessary in order to permit undisturbed action as a simple beam. Even with less separation than this, it is likely that the predominating action would be that of a simple, rather than a fixed, beam, and in this case it would be expected that the crack would be at the

[41] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 122.
[42] *Loc. cit.*, March, 1931, Papers and Discussions, p. 432.
[43] *Loc. cit.*, p. 433.
[44] *Loc. cit.*, May, 1928, Pt. 3, p. 122.

center of the span. Unfortunately, the writer cannot throw any light on the apparent contradiction between the evidence of Fig. 80 and that of Figs. 130 to 133, inclusive. These diagrams show the results of the test as far as they are known.

Dr. Kalman refers[45] to an alleged statement of Section 32 "that the arches undergo an appreciable lengthening under the effect of a load." The writer does not find such a statement in Section 32, or elsewhere, nor does he recall any basis for such a statement.

The writer claims nothing for his analysis of the effect of the taper of the dam on the applicability of the ordinary methods of computing flexural stress, except that: (1) The assumptions used seem to meet the conditions of the structure; (2) that, as shown in Fig. 106, the results agree reasonably well with the results of the test and with those found by the late Professor Cain whose attack of the problem was from a different direction; and (3) that the two analyses agree in indicating that the effect of the taper in this structure was small. The coefficients given by Dr. Kalman, are for angles of taper larger than any in the Stevenson Creek Dam, but it seems as if the results for his analysis would not differ greatly from those found by Professor Cain or by the writer.

The writer regrets to find that he failed to state that in plotting strains measured by the telemeters for comparison with strains measured by radius meter, clinometer, and strain gauge, the telemeter strains were first corrected to strains at the surface of the dam. The correction was made on the assumption that sections originally plane remained plane. This may help to answer some of Dr. Kalman's questions[46] on Chapter *K*. His uncertainty, however, as to whether the strains are represented by his Equations (161) or (162), may be settled by reference to the writer's statement[47] that,

"Generally, however, the strains plotted in the diagrams of this report are the observed strains, and in order to determine the stresses with exactness from the given strains, it is necessary to take into account Poisson's ratio and the strains mutually at right angles with each other."

Dr. Kalman quotes the writer as stating,[47a]

"That the equivalent load producing bending causes moments to occur even in the center at Elevation 50, despite the fact that a crack is known to exist at this point."

What the writer did say[48] is that "it was carrying load [not moment] even at the crack for all elevations shown in Fig. 124[49] except at Elevation 60." The presence of a crack might disqualify the structure for carrying bending moment, but does not prevent its carrying load. A cantilever beam may carry load even at the free end where there is no bending moment. However, it is even possible that there should be horizontal bending moment as well as load, at the vertical crack at the 60-ft. elevation. If the concrete was in direct

[45] *Proceedings*, Am. Soc. C. E., March, 1931, Papers and Discussions, p. 438.
[46] *Loc. cit.*, p. 440.
[47] *Loc. cit.*, May, 1928, Pt. 3, p. 162.
[47a] *Loc. cit.*, March, 1931, Papers and Discussions, p. 442.
[48] *Loc. cit.*, p. 172.
[49] Incorrectly given in the report as Fig. 54.

compression, as it would be when the crack closed with the filling of the reservoir (see Fig. 77), there could be a bending moment at the crack without producing tension.

There seems to be no justification for using the weight of the dam in attempting "to explain the presence of compressive stress at Elevations 4 and 10,"[50] since there was no change in the weight of the dam between the taking of the no-load readings just before the filling of the reservoir, and the taking of the load readings just after the filling of the reservoir.

The supposition that the vertical crack near the bottom may have "occurred while the water head was being changed from 60 to 50 feet between experiments," is not consistent with the fact as stated by the writer[51] that the crack occurred between 4:45 and 10:45 P. M. of the day of the first complete filling of the reservoir while the head was being held at 60 ft.

When the writer was delegated to the project of carrying out the test of the Stevenson Creek Dam, the prospect of securing results of value appeared as a forlorn hope. Even if accurate data could be secured, the hope of solving the arch dam problem by means of a single test seemed "a large order," and the prospect of raising money to carry out many tests at upwards of $100 000 each seemed even less than a forlorn hope. The writer had had considerable experience with making field tests of reinforced concrete structures under load. In these tests it had been found almost invariably, that the strains measured under loads that were too small to crack the concrete, had so large a percentage of error that little use could be made of them. Yet here was a test structure for which practically all the strains to be measured would be smaller than that under which the concrete would crack.

In the testing of a certain reinforced concrete slab floor[52] the temperature decreased so continuously that for a considerable period the slabs deflected upward at the center as the loading progressed. Obviously, such behavior threw serious difficulties into the interpretation of the data and yet there appeared to be fully as great an opportunity for extreme variation in temperature in the test dam as in any building test that had been made.

The most important single element in surmounting the difficulties was the carrying out of each complete test within so short a time and under such conditions that there could be no appreciable temperature change and no appreciable deformation due to humidity changes. As viewed in retrospect, the adoption of this measure seems most obvious, and it may be a confession to admit that it required some time to arrive at this procedure, but such is the fact. Other important elements were the development of strain gauges with a higher degree of precision than had previously been available, and the successful development and application of a clinometer for the measurement of deflections. So closely associated with these features of the test as to be almost inseparable from them, in contributing to a successful outcome of the investigation, is the nature of the structure tested. If it had been a thinner dam of reinforced concrete, depending on the reinforcement for the tensile

[50] *Proceedings*, Am. Soc. C. E., March, 1931, Papers and Discussions, p. 443.
[51] *Loc. cit.*, May, 1928, Pt. 3, p. 122.
[52] *Bulletin 184*, Univ. of Illinois, Eng. Experiment Station, p. 78.

resistance, the occurrence of promiscuous cracking would have rendered the interpretation of the data almost impossible. If it had been made heavy enough to avoid all cracking the probability is that the loss in proportionate accuracy of the observations would have more than offset the difficulties due to the relatively small amount of cracking that did occur.

The Committee of Engineering Foundation on Arch Dam Investigation is to be congratulated on providing a reservoir so small that it could be filled in a short time with the water supply available, and of such design that it was possible to make the most of the data secured.

It is true that the test of the dam has not solved the arch dam problem; but the usefulness of the data in forming a criterion for judging the value of model tests and the soundness of the various methods of analysis, such as those presented by Messrs. Cain, Carlson, Savage and Houk, Smith, and Westergaard, is greater than the writer visualized in his most optimistic moments prior to the completion of the tests.

The writer indulges in no illusion that the Arch Dam Committee and the U. S. Bureau of Standards planned and carried out the test to provide him with the most interesting and stimulating two years of work that he had ever experienced, but even if it had been planned for this purpose, it is difficult to understand how it could have been more successful. The value of the enthusiastic support of all members of the testing crew[53] throughout has been entirely out of proportion to their prominence in the report, and the writer wishes again to acknowledge his indebtedness to their assistance, especially that of Messrs. R. W. Carlson and Inge Lyse.

ALFRED D. FLINN,[54] M. AM. SOC. C. E. (by letter).[54a]—On deterioration of concrete, the statement by Mr. Kinney, is, of course, a contribution of value because of his background and his familiarity with the subject. The necessarily brief Part IX of Vol. I of the Arch Dam Committee's report was not written hastily, nor without basis of extended experience and consultation with other engineers. Portland cement and its use in hydraulic structures constitute too large a subject to be treated in the reports of the Arch Dam Committee or in this discussion. Able investigators have devoted attention to this subject, and references are made in the following paragraphs to a few of their published papers.

The intent of the previous statement,[55] and of this one, is constructive—to safeguard and increase the usefulness of one of the most important materials of engineering construction by giving currency to lessons learned by the experience and research of a number of engineers, so as to reduce unfortunate consequences of unintelligent misuse, and indicate lines along which improvement of practice may proceed. The writer's remarks were directed not to cement alone but to cement as incorporated in concrete. He heartily endorses Mr. Kinney's statements to the effect that concrete for hydraulic works must be well made beginning with the selection of the aggregates, and so well

[53] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 271.
[54] Secy., United Eng. Trustees, Inc., and Director, The Eng. Foundation, New York, N. Y.
[54a] Received by the Secretary April 27, 1931.
[55] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 267, Pt. IX.

placed and cured as to be as impervious as practicable, with the best of knowledge, skill, and means; but the cement demands more critical attention than was formerly, and still often is, given it. That tests for cement hitherto in vogue are inadequate for certain uses has been demonstrated convincingly. For some purposes, it may even be necessary to study the raw materials and their geologic history.

In alluding to different cements, in the report, the writer had in mind not so much contrast of so-called standard Portland cement with other types of cement as variation among different lots of Portland cement. In cements as in other substances there is evidence of differences that elude even careful chemical analysis as usually practised. It appears to be important that for the more exacting conditions of hydraulic structures engineers should have the aid of adequate methods and means for the selection of dependably suitable lots of cement. It would further appear to be wholly to the advantage of all concerned to have effective co-operation of cement manufacturers, engineers, and scientists in persistent study of the problem until a sufficient solution is achieved. That cements and concretes have been improved and important additions made to recorded information, is gratefully acknowledged, but research and observation of experience should go on.

Concrete made with separate lots of cement, even from the same mill, that have passed quite satisfactorily the specifications of the American Society for Testing Materials, has acted differently despite the fact that conditions of use and exposure were identical, within the degree of refinement of good engineering and construction practice—same testers in same laboratory, same transportation and storage, same water and aggregates, same mixing and placing equipment, methods, and men, same curing conditions, same exposure in adjacent portions of same structures. More than one observant engineer who has seen his structures months or years after completion, has noted differences of more or less importance.

The writer had been informed that there is no insoluble combined lime, excepting possibly calcium fluoride. Therefore, if water percolates through concrete, solution will occur and will continue so long as there is lime in the aggregates or in the cement of those portions of the concrete in contact with the water, and the strength of the concrete will continue to decrease, as has been demonstrated by experiments for the Board of Water Supply of New York and field observations of structures; hence the importance of imperviousness as a quality of concrete for hydraulic structures. This factor should have diligent attention by competent persons in the design, construction, and maintenance of dams and other structures.[56]

In discussing engineering structures, time qualifiers are relative. When considering durability of dams, the writer is thinking of long periods; rates of action that would be very slow in some processes would be relatively quick in the life of a dam. Beyond avoidance of possibility of actual failure, a dam up stream from a populous district should not develop, even after many

[56] See, also, "Corrosion of Concrete," by John R. Baylis, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 791.

years, any appearance which could arouse even unwarranted suspicion of its security; hence the writer's endeavor to impress upon engineers, contractors, and inspectors the necessity for great care in every step.

"Acceptable" as applied to cement in Part IX of the report means permissible for use at the time and under the conditions of construction of a given dam, with the other ingredients selected for the concrete, and with consideration for the expected service conditions following completion. It should not be confused with "accepted," meaning have satisfied a specification or series of tests.

When writing the report, there was no thought of suggesting special cements or unusual methods. The need is to get all persons who attempt hydraulic structures of Portland cement concrete to comprehend the conditions which these structures must withstand and to realize the inexorable demand for faithfulness in design, construction, and inspection, including the selection of all materials—not in general, but lot by lot as used.

Conditions to which many hydraulic structures are exposed, and particularly thin dams, are much more severe, over a long period of years, than some engineers and constructors apprehend, who from the nature of their work usually see only new structures. Dams are commonly intended to have great permanence. Their security must be unquestionable, for upon it ordinarily depend, directly or indirectly, lives of human beings, properties of great value, and extensive investments.

Engineering thinking and practice would be improved if engineers should use "concretes" in the plural in many statements, just as they do "steels" and "irons." This usage would help every one to keep in mind the fact that there are many concretes in the field of Portland cement alone. No competent engineer would use one kind of iron or of steel for every purpose.

When submitted to more elaborate tests, cements which "pass the standard specifications" are bound to differ, from mill to mill, from time to time as shipped from any given mill, and due to variations in periods and conditions of storage—even within the group called "normal Portland cement." Experience has taught that an engineer who aims at the highest quality in his structures over several years in service cannot just buy "Portland cement." For some kinds of work but little discrimination is worth while; for other kinds very careful selection may be required, based on more information than is given by "standard" tests.

If all Portland cements were strictly alike, they would re-act alike under like conditions. That they do not has been observed more than once and has been demonstrated for certain conditions by Mr. Dalton G. Miller, as reported in his paper "Resistance of Portland Cement Concrete to the Action of Sulfate Waters as Influenced by the Cement." [57]

Since it appears to be commercially impracticable to make cement alike, to the degree intended herein, and since requirements are various, is it not desirable to give engineers more complete information, when needed, so that selection can be made according to the degree of discrimination demanded by conditions of the job in hand?

[57] *Proceedings*, Am. Soc. for Testing Materials, Vol. 28, Pt. II, 1928, p. 448, and discussion.

The matter of variations in standard Portland cements was discussed[58] at length by Dr. P. H. Bates as Author-Chairman in his report of Committee 202 of the American Concrete Institute. This report of Dr. Bates was accompanied by a discussion by Thaddeus Merriman, M. Am. Soc. C. E., also a member of Committee 202. Dr. Bates in his introduction used the following significant words:

"It is felt also that the discussion will dispel the erroneous idea that there is such a thing as a standard Portland cement. It is true that all manufacturers of the commodity make an especial effort to have it pass a certain set of standard requirements, but in so doing it has resulted that not only does the product of the majority of mills meet these at all times but the product may exceed these requirements to various degrees at any time. It is further believed that this discussion may serve to indicate that Portland cement does develop other properties than those of hardening and acquiring strength. It is hoped that it will be made clear that some properties, as volume changes and heat developed during the reaction with water, may be so paramount in special cases as practically to force the ignoring of strength."

Mr. Merriman, in his discussion, indicated that all Portland cement is a mixture of different cementing constituents and that the qualities of a particular cement depend on the proportions in which these constituents happen to be present. This phase of the question was also presented at considerable length by Mr. Merriman in his Paper No. 308, on "Cement," before the World Engineering Congress of 1929.

So important is the subject, and of such widespread concern, that a substantial sum might advisedly be expended for the good of the public by the proper governmental agency (the National Bureau of Standards, preferably), utilizing and supplementing the extensive field investigation of the durability of concrete being conducted by the American Concrete Institute with the co-operation of the Portland Cement Association and owners of hundreds of structures, in further, fundamental, scientific investigation of the durability of concrete and steps to be taken to assure durability when designing and building concrete structures.

In conclusion and to forestall misunderstanding, the writer would add that there are very many concrete structures of advanced age that are sound and give good promise of so continuing indefinitely.

As to earthquakes and ice pressure, there is nothing to add other than an acknowledgment of Mr. Wood's suggestion of the possibly large "water-hammer" effect on a dam, of the water in the reservoir during earthquake vibration[59] and a general reference to the efforts, in recent years, of Japanese and American engineers and of scientists in several countries, to add to knowledge of earthquakes and to improve engineering practice by the application of all available information.

[58] *Proceedings,* Am. Concrete Inst., November, 1929.

[59] *Proceedings,* Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2609.

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DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion

BY MESSRS. THOMAS C. SHEDD, DAVID M. WILSON, AND MARSHALL G. FINDLEY

THOMAS C. SHEDD,[21] M. AM. SOC. C. E. (by letter).[21a]—The writer has had the good fortune to be closely associated with Professor Cross since the fall of 1922 when he first conceived the fundamental idea and began the development of his method of "moment distribution". Some inkling of the extent to which the author has developed this method has been given by several discussions contributed by his former students. More significant, it seems to the writer, is the number of discussers who, presumably unfamiliar with the method prior to the publication of the author's paper, have so quickly grasped the fundamental procedure as to be able to see at once its far-reaching utility, and, in many cases, to present discussions clearly illustrating applications of which the author little more than hinted. In view of the extremely condensed presentation this seems a remarkable tribute to the simplicity and directness of this new tool of analysis.

The development of the method and the manner of its presentation show that the author has strived to eliminate the academic "fiddle-faddle" and unnecessary complexity with which many of the classical methods of statically indeterminate analysis have always been embellished, and to change these from mathematical playthings for academicians to working tools for engineers.

Moment distribution by successive approximations is equally useful whether the engineer needs only a rough approximation or a precise analysis. One who uses the method frequently will be surprised at the number of cases in which

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in May, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin and E. F. Bruhn; October, 1930, by Messrs. A. H. Finlay, R. F. Lyman, Jr., R. A. Caughey, Orrin H. Pilkey, and I. Oesterblom; November, 1930, by Messrs. Edward J. Bednarski, S. N. Mitra, Robert A. Black, and H. E. Wessman; January, 1931, by Messrs. Jens Egede Nielsen, F. E. Richart, and William A. Oliver; February, 1931, by Messrs. R. R. Martel and Clyde T. Morris; March, 1931, by Francis P. Witmer, M. Am. Soc. C. E.; and May, 1931, by Messrs. T. F. Hickerson, F. H. Constant, W. N. Downey, and E. C. Hartman.

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[21a] Received by the Secretary, April 8, 1931.

an understanding of its fundamental principles enables him to make a mental estimate of indeterminate effects, which is sufficient for his purpose, and at the number of other cases in which a single distribution of the fixed-end moments yields adequate information; and, yet, the same procedure which so quickly gives an approximate answer, with the expenditure of very little additional labor (far less than is required for any kind of answer in other methods), will result in a solution of any desired precision.

Appreciation of the power and utility of Professor Cross' method so increases with longer acquaintance that one is tempted to offer a multiplicity of examples of its application. The number of excellent discussions already presented makes this unnecessary, and the writer will content himself with expressing the opinion that "moment distribution" is the most powerful and versatile tool presented for the use of structural engineers in many years. In combination with the author's paper entitled "Column Analogy"[22] this paper will exert an increasingly widespread influence on thinking and design in the structural field. Finally, for the benefit of those who lament that "moment distribution" is not a collection of generalized formulas, the writer would like to quote from Percival Lowell:[23]

"But worse still for their employed symbols tend to fictitious understanding. standing. Formulæ are the anæsthetics of thought, not its stimulants; * **

DAVID M. WILSON,[24] ASSOC. M. AM. SOC. C. E. (by letter).[24a]—The author has described a method of analysis which is of special interest to one familiar with the slope-deflection method of analyzing statically indeterminate frames. In fact, the steps used in distributing fixed-end moments are based directly on the slope-deflection equation, and as Professor Cross states, the method is, therefore, not approximate, but a method of successive approximations. As the number of approximations is increased, the results approach nearer and nearer those obtained by rigorous application of the slope-deflection method. It should be noted, however, that in the slope-deflection method no account is taken of the change in length of any member due to axial stress. To this extent, both the slope-deflection method and the method of distributing fixed-end moments are approximate.

In the example presented by Professor Cross, joint displacement, due either to horizontal forces or to unsymmetrical vertical loads, has been disregarded. The writer wishes to submit a direct procedure which he believes is a general solution of the problem.

The writer prefers to use the same convention for signs of moments as ordinarily used in the slope-deflection method; that is, clockwise moments at the ends of members are positive and counter-clockwise moments are negative. The general slope deflection equation,

$$M_{AB} = 2EK\,(2\theta_A + \theta_B - 3R) \pm C \ldots\ldots\ldots\ldots (76)$$

[22] "The Column Analogy," by Hardy Cross, *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[23] "Mars and Its Canals," Preface, p. IX.

[24] Assoc. Prof. of Civ. Eng., Univ. of Southern California, Los Angeles, Calif.

[24a] Received by the Secretary April 21, 1931.

expressed in the form,

$$M_{AB} = 2E\left(2\frac{I}{L}\theta_A + \frac{I}{L}\theta_B - 3\frac{I}{L^2}d\right) \pm C \ldots\ldots\ldots\ldots (77)$$

suggests four steps for a general solution, as follows:

I (*a*).—Compute the fixed-end moments at the ends of all members.

I (*b*).—Compute and properly distribute the moments at the ends of the columns, due to their deflection, before rotation of the joints is assumed to have taken place.

I (*c*).—Add the moments obtained in Steps I (*a*) and (*b*) and place these values at the ends of members in a line diagram of the frame.

II.—Distribute the unbalanced moment at each joint of the frame so that the sum of the moments at each joint is equal to zero.

III.—"Carry over." In Steps II and III, rotation of the joints is assumed to have taken place.

IV.—Correct the column moments so that the sum of all moments acting on the columns is equal to zero. The distribution of unbalanced column moments is made in the same way as in Step I (*b*), and for the same reason.

The sum of the values found by Steps I, II, and III, for the girders and Steps I, II, III, and IV for the columns is the first approximate value of the moments sought. Better values may be obtained by repeating Steps II and III for the girders and Steps II, III, and IV for the columns, two or more times. Usually, three cycles of operations will give results which are sufficiently accurate for all practical purposes.

The foregoing steps can best be explained by examples selected for the purpose. Steps I (*a*), II, and III have been described by Professor Cross and further explanation of them is not necessary. Three examples are worked out completely, illustrating particularly Steps I (*b*) and IV, and the results, after three cycles of operations, are compared with those obtained by the slope-deflection method (see Table 15).

TABLE 15.—COMPARISON OF MOMENTS DETERMINED BY DISTRIBUTING FIXED-END MOMENTS WITH THOSE BY SLOPE-DEFLECTION METHOD.

Item	Description	Moments at Joints Indicated, in Thousands of Inch-Pounds					
		M_{DA}	M_{AD}	M_{AB}	M_{BA}	M_{BC}	M_{CB}
	Example 1						
1	Slope-deflection method..	−551.51	−127.27	+127.27	+200.00	−200.00	−321.21
2	By Fig. 44.....	−550.50	−126.80	+125.00	+197.00	−201.50	−321.20
	Example 2						
3	Slope-deflection method..	+7.55	+18.14	−18.14	+16.14	−16.14	−9.55
4	By Fig. 45.....	+7.34	+17.68	−18.49	+16.49	−15.68	−9.34
	Example 3						
5	Slope-deflection method..	−50.00	−33.33	+33.33	+16.67	−16.67	−16.67
6	By Fig. 46.....	−49.85	−34.23	+31.83	+16.79	−15.78	−16.17

Example 1.—The frame for this example and its solution, are shown in Fig. 44. The columns are assumed of equal length and the relative stiffnesses, $\frac{I}{L}$, of the various members are as shown in circles.

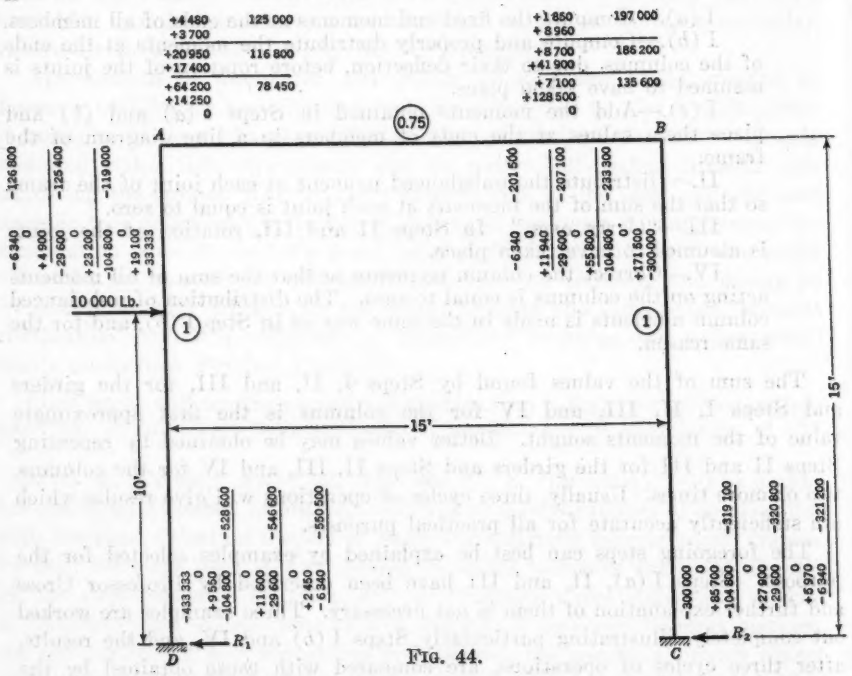

FIG. 44.

In Step I (a), fixed-end moments, in inch-pounds, are $M_{DA} = -133\,333$; $M_{AD} = +266\,666$; and $M_{AB} = M_{BA} = M_{BC} = M_{CB} = 0$. In Step I ($b$), the algebraic sum of all moments acting on the columns must be equal to zero. Therefore,

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} = -1\,200\,000 \ldots\ldots\ldots\ldots (78)$$

Before rotation of the joints is assumed to have taken place, the following moments exist at the ends of the columns, due to their deflection. The deflections, d (slope-deflection notation), for Columns AD and BC are equal:

$$M'_{AD} = M'_{DA} = \frac{-3\,I_{AD}\,d(2E)}{L^2_{AD}} \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

and,

$$M'_{BC} = M'_{CB} = \frac{-3\,I_{BC}\,d\,(2E)}{L^2_{BC}} \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Therefore,

$$\frac{M'_{AD}}{M'_{BC}} = \frac{I_{AD}\,L^2_{BC}}{I_{BC}\,L^2_{AD}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

In this example, $I_{AD} = I_{BC}$ and $L^2_{BC} = L^2_{AD}$. Hence, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB}$. Distribute the moment, $-1\,200\,000$ in-lb., to the ends of the columns

so that, $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -1\,200\,000$ in-lb. Then, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = -300\,000$ in-lb.

In Step I (*c*) by combining the moments obtained in Steps I (*a*) and (*b*) the initial values, in inch-pounds, are as follows: $M_{DA} = -133\,333 + (-300\,000) = -433\,333$; $M_{AD} = +266\,666 + (-300\,000) = -33\,333$; $M_{AB} = M_{BA} = 0$; and, $M_{BC} = M_{CB} = -300\,000$.

Place these values at the ends of the proper members on the line diagram of the frame (Fig. 44).

In Step II, distribute the unbalanced moment at each joint of the frame in proportion to the stiffness of the members.

In Step III, "carry over."

In Step IV, the sum of the column moments is found to be:

$$(-433\,333 + 0 + 9\,550) + (-33\,333 + 19\,100 + 0)$$
$$+ (-300\,000 + 171\,500 + 0) + (-300\,000 + 0 + 85\,700) = -780\,800$$

which does not agree with the value of $-1\,200\,000$ in-lb. found in Step I (*b*).

Distribute the unbalanced moment, $-1\,200\,000 - (-780\,800) = -419\,200$ in-lb., to the ends of the columns so that $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -419\,200$ in-lb., in which M'_{AD}, M'_{DA}, etc., are the correction moments at the ends of the columns, due to deflection.

For the same reasons as in Step I (*b*), $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = \frac{419\,200}{4} = -104\,800$ in-lb. The first approximate values of the moments, in inch-pounds, are: $M_{AD} = -119\,000$; $M_{DA} = -528\,600$; $M_{AB} = 78\,450$; $M_{BA} = 135\,600$; $M_{BC} = -233\,300$; and $M_{CB} = -319\,100$.

Repeat Steps II and III for the girder and Steps II, III, and IV for the columns, until the desired accuracy is obtained.

Example 2.—In this example (Fig. 45), the moments of inertia of all members are assumed equal and the relative stiffnesses are shown in circles, as in Example 1.

In Step I (*a*), the fixed-end moments for the girder are -40 at A and $+20$ at B; and zero at both ends for each column.

In Step I (*b*), there are no horizontal forces acting on the columns so that the sum of the column moments is equal to zero, before rotation of the joints is assumed to have taken place. Therefore, no distribution of moments is required at this point.

Steps II and III are carried out as usual.

In Step IV, the sum of the column moments is found not equal to zero, because of the deflection of the frame caused by the unsymmetrical loading; that is, $(0 + 16 + 0) + (0 + 0 + 8) + (0 - 8 + 0) + (0 + 0 - 4) = 12$. Obviously, the unbalanced moment is -12, and this value is distributed to the ends of the columns so that, $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -12$, as in Example 1, Steps I (*b*) and IV. The correction moments due to deflection are, for the same reasons, as in Example 1, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = -3$. The first approximate values of the moments are determined next, and the prescribed cycle of operations is repeated.

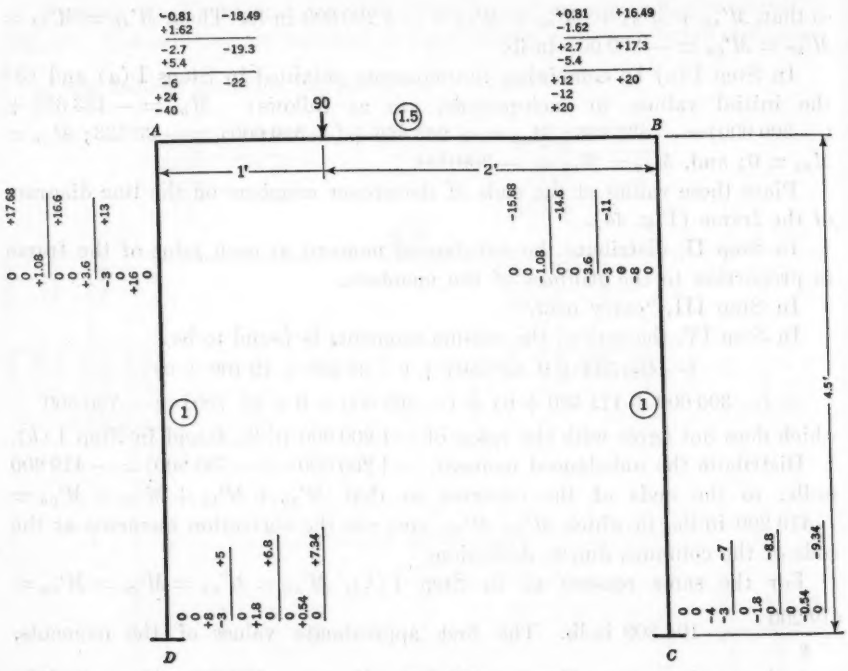

FIG. 45.

Example 3.—The frame for this example (Fig. 46), is selected to show how the four steps solve the problem of an unsymmetrical bent. The moments of inertia of all members are assumed equal.

In Step I (*a*), all fixed-end moments are zero.

In Step I (*b*), the algebraic sum of all moments acting on the columns must be equal to zero. Therefore,

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} + R_1 \times 10 + R_2 \times 20 = 0 \ldots\ldots (82)$$

Furthermore,

$$R_1 + R_2 = 10 \ldots\ldots (83)$$

and,

$$M_{BC} + M_{CB} + R_2 \times 20 = 0 \ldots\ldots (84)$$

Eliminating R_1 and R_2 from Equation (82),

$$M_{AD} + M_{DA} + \tfrac{1}{2}(M_{BC} + M_{CB}) = -100 \ldots\ldots (85)$$

Before rotation of the joints is assumed to have taken place, the moments in Equations (79), (80), and (81) exist at the ends of the columns, due to their deflection. The deflections, d, for Columns AD and BC are equal.

In this example, $I_{AD} = I_{BC}$ and $L^2_{BC} = 4\,L^2_{AD}$. Hence, by Equation (80), $M'_{AD} = 4\,M'_{BC}$.

Distribute the moment, -100, to the ends of the columns so that $M'_{AD} + M'_{DA} + \frac{1}{2}(M'_{BC} + M'_{CB}) = -100$. Then, $M'_{BC} = M'_{CB} = \frac{-100}{9} = -11.1111$, and, $M'_{AD} = M'_{DA} = 4 \times -11.1111 = -44.4444$.

In Step I (*c*), since all the fixed-end moments in Step I (*a*) are zero, the values obtained in Step I (*b*) are placed at the ends of the columns on the line diagram of the frame, Fig. 46.

Steps II and III are carried out as usual.

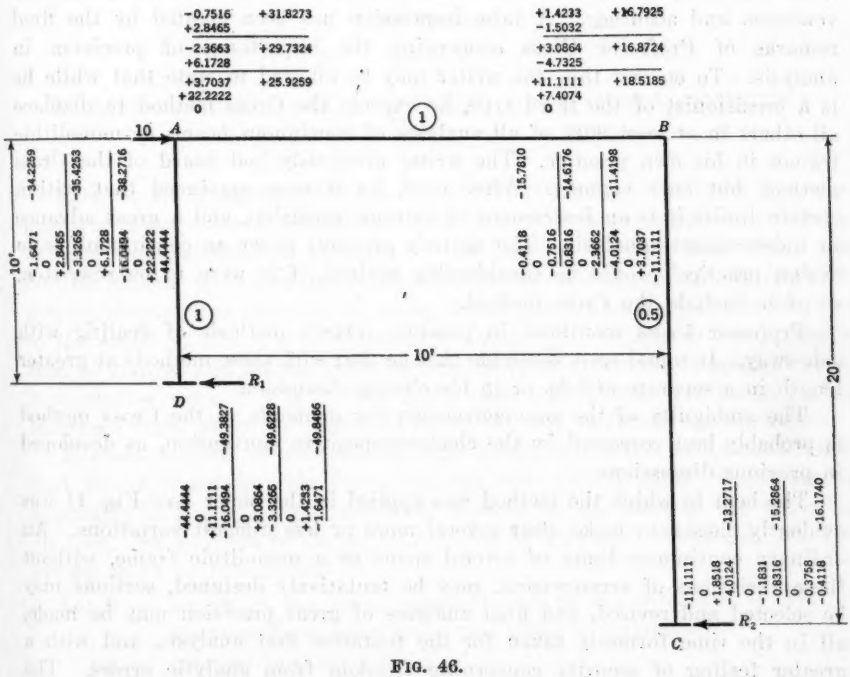

FIG. 46.

In Step IV, the moment relation, as found in Step I (*b*) (see Equation (84)), is not satisfied, since $-22.2222 - 33.3333 + \frac{1}{2}(-7.4074 - 9.2593) = -63.8888$. Distribute the unbalanced moment, $-100 - (-63.8888) = -36.1112$, to the ends of the columns so that, $M'_{AD} + M'_{DA} + \frac{1}{2}(M'_{BC} + M_{CB}) = -36.1112$, in which M'_{AD}, M'_{DA}, etc., are the correction moments at the ends of the columns, due to deflection.

For the same reasons as in Step I (*b*), $M'_{BC} = M'_{CB} = \frac{-36.1112}{9} = -4.0124$; and, $M'_{AD} = M'_{DA} = 4\ M'_{BC} = -16.0494$.

The first approximate values of the moments are as follows: $M_{AD} = -38.2716$; $M_{DA} = -49.3827$; $M_{AB} = 25.9259$; $M_{BA} = 18.5185$; $M_{BC} = -11.4198$; and $M_{CB} = -13.2717$. Repeat Steps II and III for the girder and Steps II, III, and IV for the columns, until the desired accuracy is obtained.

In the three examples presented, all members are assumed to have uniform cross-section throughout their lengths and, therefore, the "carry-over" factor is $+\frac{1}{2}$. This is by far the most common case. If, however, a member has a variable cross-section between joints, the proper "carry-over" factor can be readily determined by the principle of area moments.

MARSHALL G. FINDLEY,[25] ASSOC. M. AM. SOC. C. E. (by letter).[25a]—The Cross method has been tested by the writer for several cases, and the results have been compared with those obtained by other methods, as to speed, convenience, and accuracy. A false impression has been created by the final remarks of Professor Cross concerning the importance of precision in analysis. To correct this, the writer may be allowed to state that while he is a precisionist of the third type, he expects the Cross method to displace all others in at least 90% of all analyses of continuous beams or monolithic frames in his own practice. The writer previously had heard of the Cross method, but only vaguely. After trial, he is now convinced that within certain limits it is an instrument of extreme precision, and a great advance in indeterminate analysis. The writer's previous paper on continuous beam design practice[26] would be considerably revised, if it were to be rewritten, so as to include the Cross method.

Professor Cross mentions, in passing, certain methods of dealing with side-sway. It would seem desirable that he deal with these methods at greater length in a separate article, or in his closing discussion.

The ambiguity of the sign-convention for moments in the Cross method is probably best corrected by the clockwise-positive convention, as developed in previous discussions.

The bent to which the method was applied in the paper (see Fig. 1) was evidently chosen to make clear several more or less difficult variations. An ordinary continuous beam of several spans or a monolithic frame, without these variations of arrangement, may be tentatively designed, sections may be selected and revised, and final analyses of great precision may be made, all in the time formerly taken for the tentative first analysis, and with a greater feeling of security concerning freedom from analytic errors. The usual procedure should be to make a number of analyses for the final design, one for dead load and one for each group or span of live loads, and then to use the method of summation to determine all maxima. This procedure has often been impossible on account of time limits when using other methods, but when the Cross method is used, it will be applied to many more situations where it is needed.

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[25] Structural Designer, Harbor Commission, Milwaukee, Wis.

[25a] Received by the Secretary, May 11, 1931.

[26] *Engineering News-Record*, March 21, 1929, p. 467.

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DISCUSSIONS

TESTS OF BROAD-CRESTED WEIRS

Discussion

BY MESSRS. ERIK G. W. LINDQUIST, AND RALPH L. PARSHALL

ERIK G. W. LINDQUIST,[8] M. AM. SOC. C. E. (by letter).[8a]—If the measurements of water surface profiles in this paper had been accompanied by simultaneous measurements of pressure on the deck of the weir and within the flowing masses, as well as simultaneous measurements of the velocity distribution up stream from the weir, the paper would have been a notable contribution to the study of the flow of water over dams. It would have been possible then to analyze and discuss every conclusion from a theoretical point of view. Nevertheless, the author is entitled to the appreciation of the profession for his laborious work, which will certainly direct the way to future experiments.

If measurements of pressure had been made, the author would have found that the ordinary formula for computing the critical depth is not always applicable to a dam. This statement is made on the basis of the following analysis. In addition to the notation given by Professor Woodburn, the following must be introduced:

Let B = length of the crest, perpendicular to the direction of flow.
z = distance above some horizontal plane of reference.
p = internal pressure.
k = velocity head.
h_e = loss of head due to friction.
r = radius of curvature.
w = specific weight.

Consider a vertical section at the crest of a dam with the form illustrated in Fig. 28. The discharge is expressed by,

$$Q = B \int_0^D v_z \, d\,z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

NOTE.—The paper by James G. Woodburn, Assoc. M. Am. Soc. C. E., was published in September, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: October, 1930, by H. W. King, M. Am. Soc. C. E.; December, 1930, by Messrs. Sherman M. Woodward and David L. Yarnell; and February, 1931, by Messrs. Boris A. Bakhmeteff and D. D. Curtis.

[8] Capt., Royal Swedish Corps of Civ. Engrs.; Office Engr., Stockholm City Water Supply, Stockholm, Sweden.

[8a] Received by the Secretary, November 6, 1930.

For a section, *a-a*, Bernouilli's equation of energy is,

$$H = z_0 + h_0 + \alpha\, k_0' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

and for a section, *b-b* (assuming that the stream lines, or filaments, are perpendicular to the sections),

$$H = z_0 + z + p_z + k_z + h_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Hence, since $k_z = \frac{v_z^2}{2\,g}$, Equations (34) and (35) may be solved simultaneously to produce,

$$v_z = \sqrt{2\,g\,(h_0 + \alpha\, k_0) - z - p_z - h_e} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

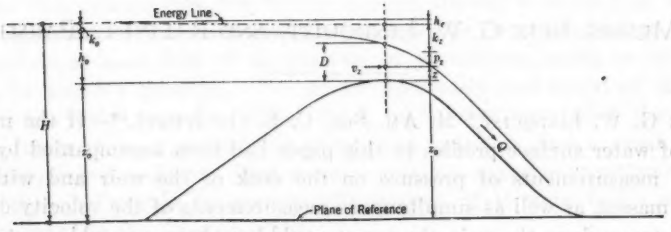


FIG. 28.

The discharge then (from Equation (33)) is equal to,

$$Q = \psi\, B\, \sqrt{2\,g} \int_0^D \sqrt{(h_0 + \alpha\, k_0) - z - p_z}\, d\,z \ldots\ldots\ldots\ldots\ldots (37)$$

in which, the coefficient, ψ (less than 1), has been inserted instead of h_e to account for friction. A pressure, p_z, is a function of the curvature of the stream line; that is, its value depends on the height of the dam and on the shape of its cross-section as well as on the head.

Assuming, first, that $p_z = D - z$ (see Fig. 28), and integrating Equation (37),

$$Q = \psi\, B\, D\, \sqrt{2\,g}\, \sqrt{(h_0 + \alpha\, k_0) - D} \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

In 1848, Belanger introduced the principle that the critical depth is that at which the discharge is a maximum for any given head. In other words, for $\frac{\partial Q}{\partial D} = 0$,

$$D_c = \frac{2}{3}\,(h_0 + \alpha\, k_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

and,

$$Q = \frac{2}{3}\,\frac{\psi}{\sqrt{3}}\, B\, \sqrt{2\,g}\,(h_0 + \alpha\, k_0)^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Next, assume that $p_z = 0$, and again integrate Equation (37):

$$Q = \frac{2}{3}\,\psi\, B\, \sqrt{2\,g} \left\{ (h_0 + \alpha\, k_0)^{\frac{3}{2}} - (h_0 + \alpha\, k_0 - D)^{\frac{3}{2}} \right\} \ldots\ldots\ldots (41)$$

Applying Belanger's principle, as before, $D_c = h_0 + \alpha\, k_0$, and:

$$Q = \frac{2}{3}\, \psi\, B\, \sqrt{2\,g}\, (h_0 + \alpha\, k_0)^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Thus, it is demonstrated that the discharge depends upon the pressure in the overfalling sheet. For equal values of energy head, the discharge is lower when the pressure has a hydrostatic distribution from the surface to the crest, that is, when $p_z = D - z$. This condition is fulfilled only on a broad-crested weir with a very smooth entrance end.

Assuming, finally, that $p_z = m\ (D - z)$, and again integrating Equation (37),

$$Q = \frac{2}{3}\, \frac{\psi\, B}{1 - m}\, \sqrt{2\,g} \left\{ (h_0 + \alpha\, k_0 - m\, D)^{\frac{3}{2}} - (h_0 + \alpha\, k_0 - D)^{\frac{3}{2}} \right\} \quad (43)$$

Once more Belanger's principle indicates that, for $\frac{\partial\, Q}{\partial\, D} = 0$,

$$\frac{3}{2}\, m\, (h_0 + \alpha\, k_0 - m\, D)^{\frac{1}{2}} + \frac{3}{2}\, (h_0 + \alpha\, k_0 - D)^{\frac{1}{2}} = 0 \quad \ldots\ldots\ldots (44)$$

which reduces to the form,

$$D_c = \frac{1 + m}{1 + m + m^2}\, (h_0 + \alpha\, k_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For D, in Equation (43), insert the value of D_c, given in Equation (45) and simplify. The result is,

$$Q = \frac{2}{3}\, \psi\, B\, \sqrt{2\,g}\, (h_0 + \alpha\, k_0)^{\frac{3}{2}} \frac{1}{\sqrt{1 + m + m^2}} \quad \ldots\ldots\ldots\ldots (46)$$

The ordinary discharge formula is,

$$Q = \frac{2}{3}\, \mu\, B\, \sqrt{2\,g}\, (h_0 + \alpha\, k_0)^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Combining Equations (46) and (47), for purposes of comparison,

$$\mu = \frac{\psi}{\sqrt{1 + m + m^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

in which, ψ is a coefficient the value of which depends upon the loss of head due to friction. Assuming that $\psi = 1$, the values of μ in Table 9 are compared with the values of D_c for the range from $m = +1$ to $m = -1$. Only when the stream filaments are parallel and straight can m equal $+1$, and m can equal zero only in a freely falling sheet of water, in which the stream filaments are concentric and in which the contours of the nappe are exposed to the air or to some other surrounding medium of constant pressure. When the contours of the escaping stream begin to converge, the internal pressure becomes higher than the pressure in the air or other medium surrounding the nappe.

TABLE 9.—COMPARISON OF CRITICAL DEPTHS

m	$\frac{D_c}{h_c + \alpha k_c}$ (Equation (45))	μ (Equation (48))	m	D_c	μ
+1.0	0.667	0.578	0.0	1.0	1.0
+0.9	0.702	0.610	−0.1	0.999	1.049
+0.8	0.738	0.640	−0.2	0.953	1.091
+0.7	0.777	0.677	−0.3	0.886	1.125
+0.6	0.817	0.715	−0.4	0.790	1.147
+0.5	0.858	0.754	−0.5	0.667	1.155
+0.4	0.898	0.802	−0.6	0.527	1.147
+0.3	0.936	0.849	−0.7	0.328	1.125
+0.2	0.968	0.898	−0.8	0.238	1.091
+0.1	0.992	0.950	−0.9	0.110	1.049
+0.0	1.0	1.0	−1.0	0.0	1.0

The linear pressure distribution given by the expression, $p_z = m\ (D - z)$, is only an assumption. However, it has proved itself applicable in some cases as will be explained subsequently.

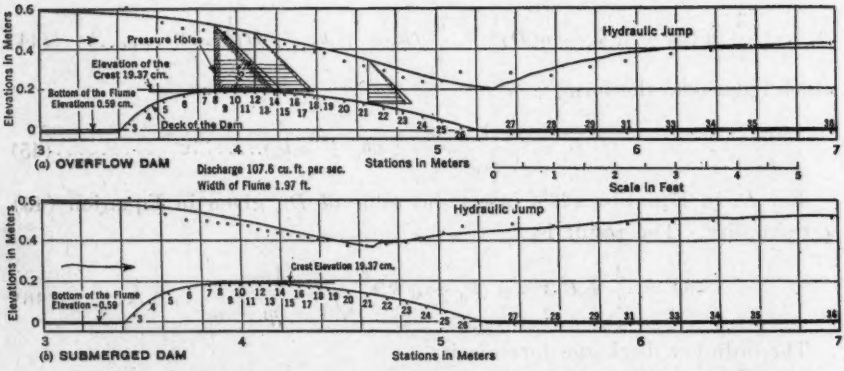

Elevations in Meters
Pressure Holes
Elevation of the Crest 19.37 cm.
Bottom of the Flume Elevations 0.59 cm.
Deck of the Dam
Hydraulic Jump
(a) OVERFLOW DAM
Discharge 107.6 cu. ft. per sec.
Width of Flume 1.97 ft.
Stations in Meters
Scale in Feet
Crest Elevation 19.37 cm.
Bottom of the Flume Elevation −0.59
(b) SUBMERGED DAM

FIG. 29.—DISCHARGE OVER A LOW WEIR, WITH SMOOTH APPROACH CHANNEL.

For the weir shown in Fig. 29, the pressure distribution, which is measured by holes in the side wall of the experimental flume, is nearly linear. The value of m for this case was 0.87 and the value of μ as computed by means of Equation (47) was 0.62. For a value of ψ equals 0.99, Equation (48) produces a value of μ equal to 0.61.

The results of measurements on weirs with a semi-circular cross-section, are as listed in Table 10.

TABLE 10.—EXPERIMENTS ON A SEMI-CIRCULAR DAM, 800 MILLIMETERS HIGH

Q, in liters per second. (1)	Total head, m. (2)	Observed pressure at the crest, m. (3)	m, observed. (4)	μ, experimental Equation (47). (5)	μ, computed by Equation (48). (6)	m, computed by Equation (58) (7)
253.4	0.361	0.172	0.69	0.643	0.666	0.70
151.7	0.262	0.139	0.78	0.625	0.627	0.78
78.7	0.159	0.092	0.87	0.606	0.607	0.87
26.2	0.085	0.052	0.93	0.594	0.598	0.93

The values of μ, computed by Equation (48) (see Column (6), Table 10) in which, ψ equals 0.98, agree very closely with those determined by Equation (47) (see Column (5) Table 10).

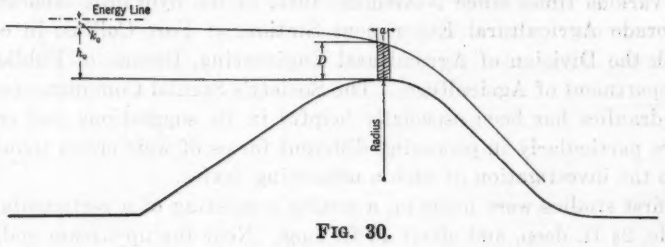


FIG. 30.

The internal pressure close to the deck at the crest can be computed approximately as follows: In Fig. 30, consider the mass element, $a\,D\,B$, in motion. The vertical forces acting are the weight of the element, $w\,a\,D\,B$, the pressure on the deck, $-w\,p\,a\,B$, and the centrifugal reaction, $-a\,D\,B\,\frac{w\,v^2}{g\,r}$. Hence,

$$w\,a\,D\,B - w\,p\,a\,B - a\,D\,B\,\frac{w\,v^2}{g\,r} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

or,

$$p = D\left(1 - \frac{v^2}{g\,r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

Since $m = \frac{p}{D}$,

$$m = 1 - \frac{v^2}{g\,r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

The mean velocity above the crest is approximately,

$$v = \sqrt{2\,g\,\frac{(h_0 + \alpha\,k_0)}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

For v, in Equation (51), insert the value of v in Equation (52) and,

$$m = 1 - \frac{2}{3}\left(\frac{h_0 + \alpha\,k_0}{r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

The values of m, in Column (7), Table 10, were computed by means of Equation (53). Although the method used is very approximate the observed and computed values are very close; but this does not prove that the method used is always applicable. As soon as the path of the particles flowing near the deck does not agree with the contour of the dam, the problem becomes very much complicated. This is the condition that exists for the dams tested by Professor Woodburn. Cushions or eddies will be found on the crest near the entrance end.

Fig. 29(*b*) illustrates that position of the hydraulic jump when the dam is submerged. As soon as this condition occurs, small waves are formed on the surface and are propagated up stream, with the result that the up-stream level is raised.

RALPH L. PARSHALL,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The study of the possibilities of measuring the rate of flow over broad-crested weirs of different types, based on the principle of critical depth, has been given attention at various times since November, 1925, at the hydraulic laboratories of the Colorado Agricultural Experiment Station, at Fort Collins, in co-operation with the Division of Agricultural Engineering, Bureau of Public Roads, U. S. Department of Agriculture. The Society's Special Committee on Irrigation Hydraulics has been especially helpful in its suggestions and criticism, and more particularly in proposing different forms of weir crests thought best suited to the investigation of such a measuring device.

The first studies were made on a setting consisting of a rectangular flume, 2 ft. wide, 2½ ft. deep, and about 10 ft. long. Near the up-stream end, on the floor of this flume, a metal convex-shaped hump was placed so that the up-stream and down-stream faces curved in an easy transition to the crest line, which was level and normal to the axis of the flume. This crest, or hump, was 10 in. high. A secondary humped crest of similar design, but only 4 in. high, was placed at a distance of 4 ft. down-stream. It was soon discovered, however, that this secondary crest was of no effect. The principal feature involved was the application of the critical depth in the formula,

$$Q = L \sqrt{g}\ H_c^{1.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

in which, Q is the rate of flow, in cubic feet per second; L, the length of crest in

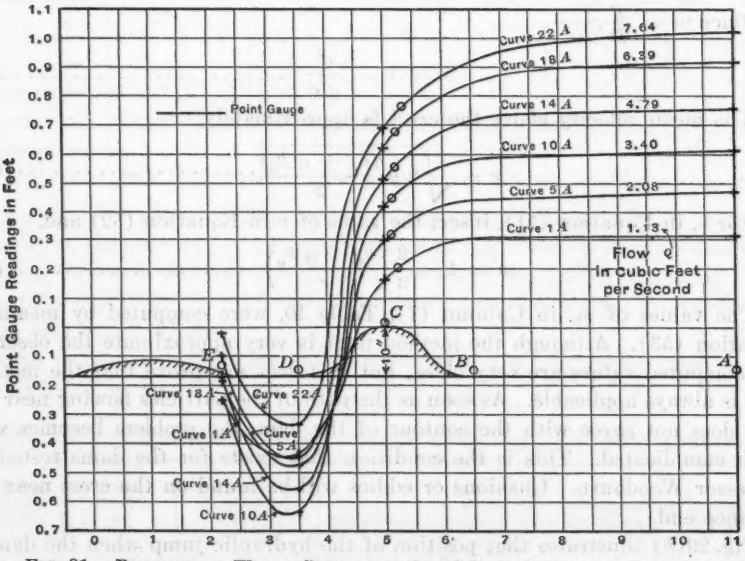


FIG. 31.—PROFILES OF WATER SURFACE WITH 10-INCH CREST AND FREE FLOW, FORT COLLINS LABORATORY, NOVEMBER, 1925.

[9] Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

[9a] Received by the Secretary, April 13, 1931.

feet; g, the acceleration of gravity; and H_c, the critical depth, in feet. From a theoretical standpoint, this critical depth should occur at the crown or crest of the hump, but the preliminary investigation of this setting indicated that the location of this particular depth was not over a fixed point.

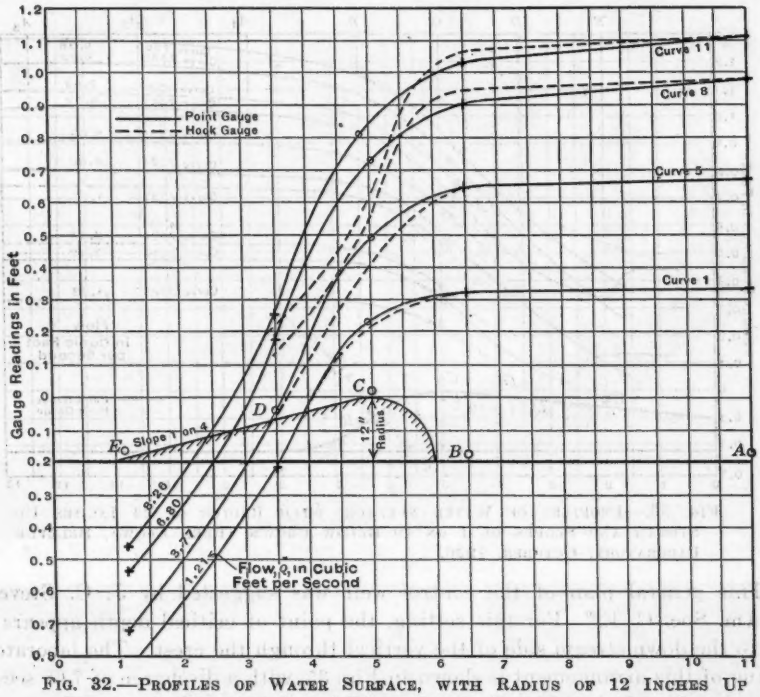


FIG. 32.—PROFILES OF WATER SURFACE, WITH RADIUS OF 12 INCHES UP STREAM AND SLOPES OF 1 ON 4 BELOW CROWN (FREE FLOW), FORT COLLINS LABORATORY, JUNE, 1926.

The profiles of the water surface along the axis of the flume with the double hump (Fig. 31) show, for a limited number of tests, that this point of critical depth shifts up stream from the vertical through the crest as the rate of discharge increases. This arrangement was changed and a single crest substituted, having a radius of 12 in. up stream, with a slope of 1 on 4 inclined downward from the crown. It will be noted in Fig. 32 that the actual location of the point of critical depth substantially agrees with the vertical through the crest.

In October, 1926, experimental apparatus consisting of a flume similar to that just described, having a single crest, 13 in. high, placed about 5 ft. down stream from the upper end of the channel, was installed at the laboratory in Bellvue, Colo. The up-stream face of this weir was of an ogee shape, in section, curved from the floor line with a radius of 31 in., with a transition from a concave to a convex surface, having a radius of 24 in. Just down stream from the crest line it was made tangent to a plane surface, inclined downward with a slope of 1 on 10. For this setting, under a free-flow discharge, the point of

critical depth shifted down stream as the rate of discharge increased. The profiles of the water surface along the axis of the flume and the location of the point of critical depth are shown in Fig. 33. This setting was altered later, to the arrangement shown in Fig. 34. The height of the crest is 13 in.

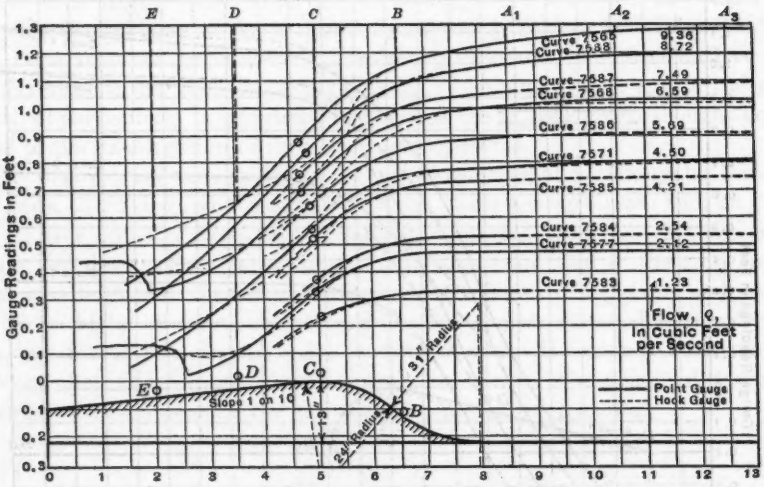


FIG. 33.—PROFILES OF WATER SURFACE, WITH RADIUS OF 24 INCHES UP STREAM AND SLOPES OF 1 ON 10 BELOW CROWN (FREE FLOW), BELLVUE LABORATORY, OCTOBER, 1926.

This general plan of the control weir was suggested by J. C. Stevens, M. Am. Soc. C. E.[10] For this setting, the point of critical depth appears to fall to the down-stream side of the vertical through the crest. The laboratory setting of this arrangement is shown in Fig. 35, with a discharge of 7.68 sec-ft.

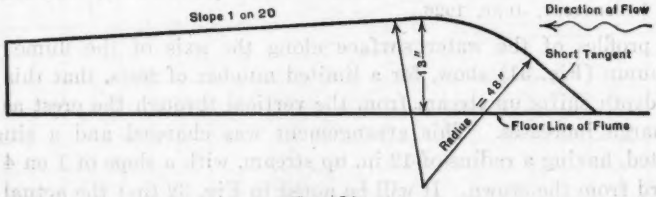


FIG. 34.

Fig. 36 is the laboratory setting for the double hump previously described (see Fig. 31). The head, as measured by the hook-gauge at the crest (where a piezometer tube to the stilling-well was set flush with the inside face of the flume wall) was consistently less than the actual depth as determined by a point-gauge. This variation in the two gauges, intended to give the head or depth over the crest, is believed to be due to the effect of cavitation or negative pressure existing just down stream from the crown of the weir. Profiles of the water surface over this type of weir are shown in Fig. 37.

[10] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 874.

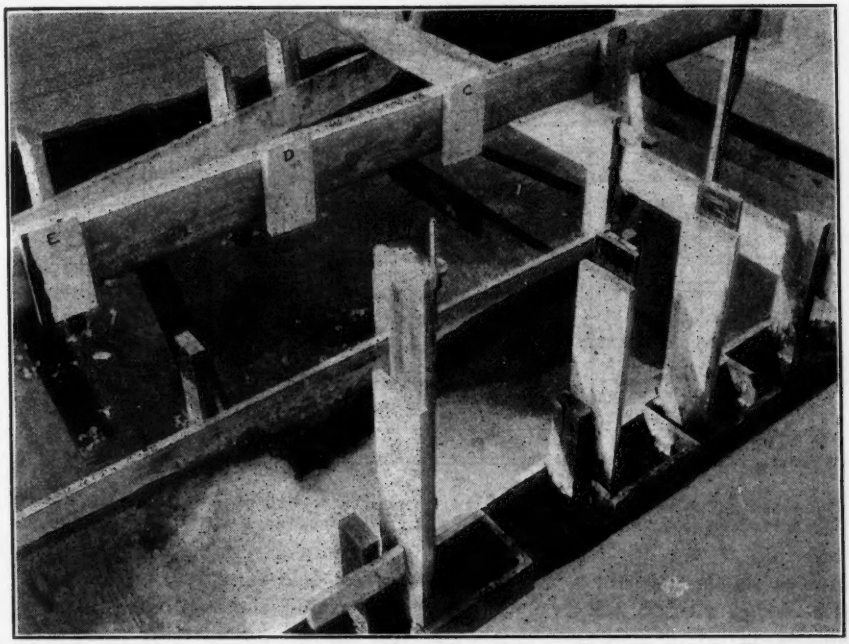


FIG. 35.—FLOW OF 7.68 SECOND-FEET OVER CREST WITH CONVEX CURVED UP-STREAM FACE (SEE FIG. 34).

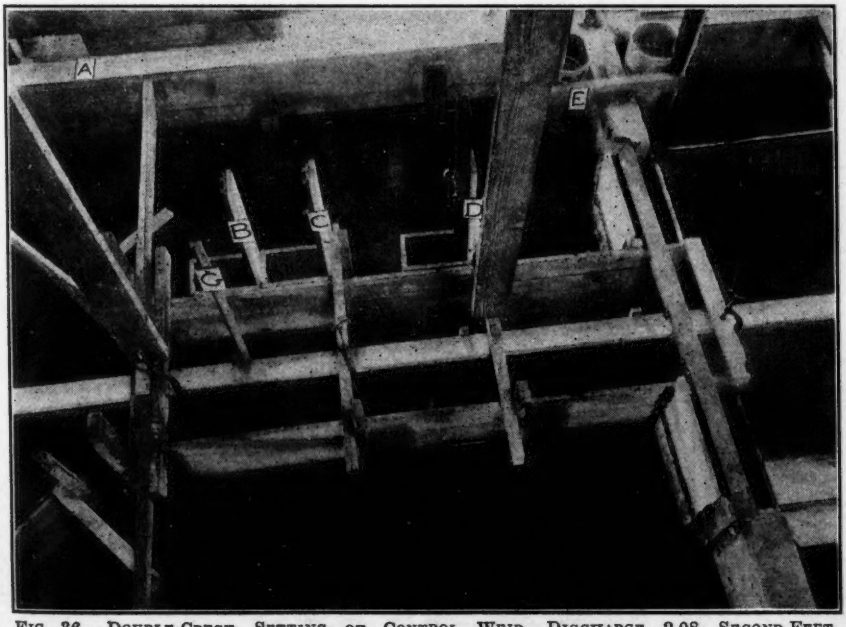
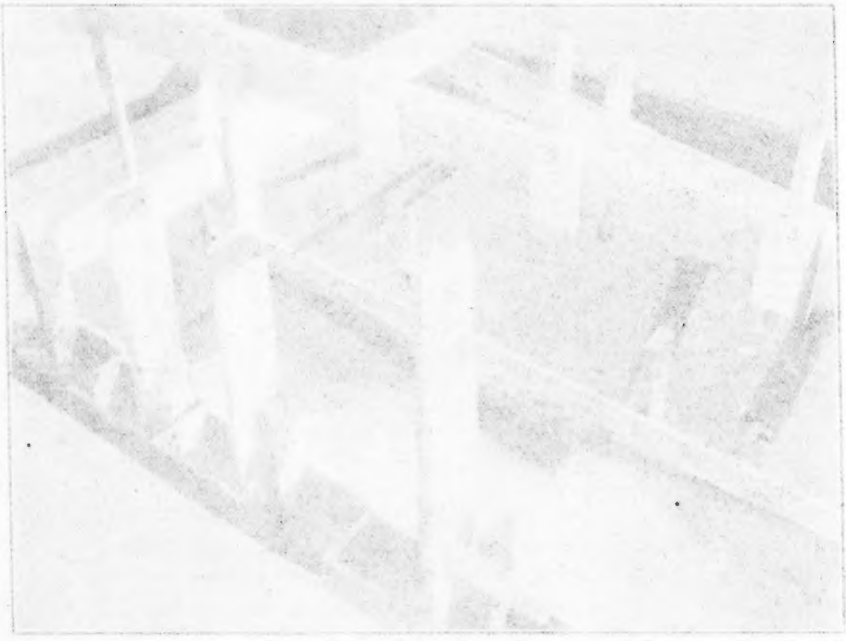


FIG. 36.—DOUBLE-CREST SETTING OF CONTROL WEIR. DISCHARGE 2.08 SECOND-FEET. POINT-GAUGE *G* LOCATING WATER SURFACE AT SECTION *B*.



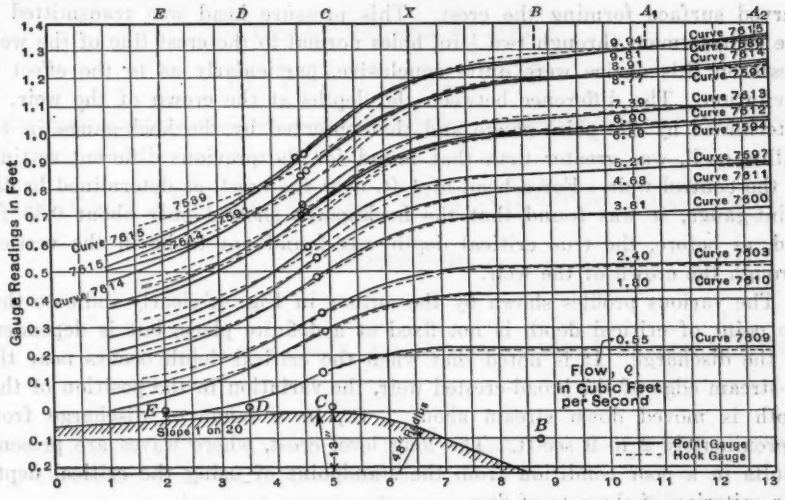


FIG. 37.—PROFILES OF WATER SURFACE WITH RADIUS OF 48 INCHES UP STREAM AND SLOPE OF 1 ON 20 BELOW CROWN (FREE FLOW), BELLVUE LABORATORY, OCTOBER, 1926.

In June, 1927, a special setting of the critical-depth flow meter, as suggested by Mr. Stevens (Fig. 38) was installed at the Bellvue Laboratory. It is essentially the same as the Herschel hollow-crest weir.[10] In this setting the pressure head on the crest, observed by the hook-gauge, was communicated to the stilling-well through a water-tight compartment placed beneath the

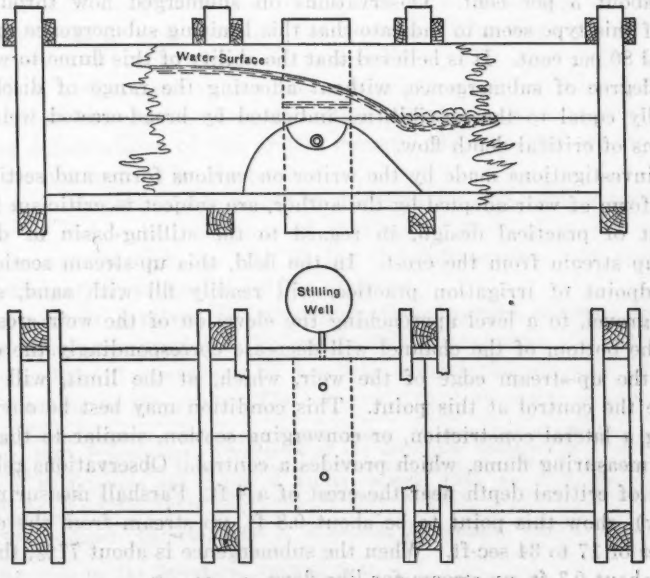


FIG. 38.—CRITICAL-DEPTH FLOW METER.

curved surface forming the crest. This pressure head was transmitted to the compartment through two ¼-in. holes normal to the crest line of the weir. Tests on this device were quite conclusive, particularly as to the effect of cavitation. The difference between the depths at the crown of the weir, as determined by the point-gauge and that observed by the hook-gauge in the stilling-well, was greater than that found for the previous different settings of the control weir. For a head of 1 ft. over the crest, as determined by the point-gauge, it was found that the hook-gauge showed only about 0.35 ft., and, as before, the true critical depth was found not to be in the vertical through the crown of the weir.

The various profiles shown by the author in Fig. 4, clearly indicate that the point of critical depth is not fixed at a definite place, but is dependent on the discharge. It is noted that when the critical depth occurs near the up-stream edge of the broad-crested weir, the variation in the position of this depth is moved down stream about 1 ft., in a range of discharge from approximately 2 to 9 sec-ft. The long level crest, where waves are present, results in a poor condition from the standpoint of using the critical depth as a criterion of the rate of flow.

Unfortunately, the data, as given in Table 1, do not show the degree of submergence for the tests thus indicated. By inspection of Fig. 4, it is estimated that for Series *D E* and *D F*, where the hydraulic jump occurs, this submergence may reach 75 to 80% as a maximum, with no material change in the coefficient, *C*. It is found that for the 2-ft. Parshall measuring flume for the head, H_a, ranging from 0.8 ft. to 1.4 ft., and with the submergence varying between 74 and 77%, the observed and computed free-flow discharges agree within about 3 per cent. Observations on submerged flow through large flumes of this type seem to indicate that this limiting submergence may equal or exceed 80 per cent. It is believed that the ability of this flume to withstand a high degree of submergence, without affecting the range of discharge, is practically equal to the possibilities indicated by broad-crested weirs under conditions of critical depth flow.

The investigations made by the writer on various forms and settings, and also the form of weir adopted by the author, are subject to criticism from the viewpoint of practical design, in regard to the stilling-basin or depressed section up stream from the crest. In the field, this up-stream section, from the standpoint of irrigation practice, will readily fill with sand, silt, and possibly gravel, to a level approaching the elevation of the weir crest. This rise in the bottom of the channel will decrease correspondingly the entrance drop at the up-stream edge of the weir, which, at the limit, will entirely eliminate the control at this point. This condition may best be obviated by providing a lateral constriction, or converging section, similar to that in the Parshall measuring flume, which provides a control. Observations relative to the point of critical depth near the crest of a 4-ft., Parshall measuring flume (free-flow), show this point to be about 0.8 ft. up stream from the crest for discharges of 17 to 34 sec-ft. When the submergence is about 77%, this point is found about 0.3 ft. up stream for like flows.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE BUTTRESSED DAM OF UNIFORM STRENGTH

Discussion

BY MESSRS. PAUL BAUMAN, I. M. NELIDOV, AND HERMAN SCHORER

PAUL BAUMAN,[24] M. AM. SOC. C. E. (by letter).[24a]—The method of designing buttressed dams as proposed by the author is well adapted to the flat-slab type, because in that case his theory holds without modification. With a multiple-arch dam, a joint between the arch barrel and the face of the buttress would be required to make it directly applicable, which is not in accordance with standard practice. The greater the span of multiple-arch dams, the greater becomes the prominence of the arch over the buttress, both as to area and moment of inertia.

In a hoof dam, for example, with spans of 200 ft., or more, between buttresses, the principle of uniform strength would scarcely prove advantageous because the greater part of the dam is in the arches, which can only be designed, safely and economically, as such. The relative load taken by the buttresses of such a dam is materially smaller than with a flat-slab dam. Even with a flat-slab dam the safe height would probably be considerably less than the one based on the author's formulas, because of stresses due to elastic deformations of the arched column which, as far as the writer is aware, are not covered by the paper.

If a dam of considerable height is designed for full load, a uniform compression of the arched columns of maximum length does not necessarily take place even though the slab during the deformation may remain practically straight, as suggested by Mr. Schorer.

Less work is involved in shortening these arched columns by bending than by concentric pressure. To enable this the columns must shorten their average radius of curvature by bending toward the air side of the dam and by opening up the joints. The latter will take place due to eccentricity of

NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in November, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: January, 1931, by Messrs. James Girand and Fred A. Noetzli; February, 1931, by Messrs. B. F. Jakobsen and Calvin V. Davis; March, 1931, by A. A. Eremin, Assoc. M. Am. Soc. C. E.; May, 1931, by Messrs. Eugene Kalman, A. Floris, Fredrik Vogt, and A. C. Schwager.

[24] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs., Consolidated, Los Angeles, Calif.

[24a] Received by the Secretary, April 6, 1931.

the line of support, primarily caused by a restraining moment at the base due to deflection of the buttress. The maximum stresses thereby produced would be materially greater than those due to uniform pressure, but would not necessarily be critical since the basic, uniform pressure could be rather low even with a strictly economical design.

A further tendency toward eccentricity in the arched columns exists during partial load, provided they are designed for full load. The resulting stresses may again be greater than those for full load, and the factor of safety may be smaller for partial load than for full load. This may sound somewhat paradoxical, but similar conclusions may be drawn from the study of structures of entirely different types, such as a continuous beam over numerous supports, etc. If designed for uniform load only, the latter may promptly fail under partial load.

To establish the principal trajectories by the author's method is more satisfactory than to determine them from the σ_x stresses (horizontal plane), assumed to be linearly distributed because, even if their distribution is nearly linear, they are more or less in error, except for a flat-slab dam with triangular buttresses. From a practical standpoint, the elimination of friction from the joints of a buttressed dam of uniform strength would probably be the outstanding task.

The author is to be congratulated for his presentation of this subject in a simple and comprehensive manner.

I. M. NELIDOV,[25] ASSOC. M. AM. SOC. C. E. (by letter).[25a]—The author's method of proportioning the buttress with the aid of the catenary curves is a process opposite to that ordinarily used. Instead of assuming loading and dimensions of a buttress and expressing stresses as functions of co-ordinates (ultimately principal stresses), the principal stresses and loadings are assumed and dimensions of the buttress are expressed as their functions.

This process is applicable only to the cases in which the continuity of loading is assured, that is, when the water level is at the crest of the dam or higher. It is applicable only for the particular loading assumed, because only with this loading is the direction of the first principal stresses at the base coincident with that of tangents to the catenary, and only for this case is there no difference as to whether or not the elementary columns are fixed or hinged at the base. If the base is fixed then for any other loading the afore-mentioned tangents must remain unchanged.

The writer wished to investigate the difference between the stress distribution computed by the author's method and by the ordinary method, by taking moments of weights and loads about the center of gravity of the base, computing moments of inertia of the base, etc. For this purpose a buttress was selected, with the following data given: $D = 100$ ft., $b = 20$ ft., $k = 0.1$ (constant, although in actual practice k increases with v, causing a "hump" at the upper part of the buttress), $\gamma_w = 62.5$ lb. per cu. ft., $\gamma_c = 144$ lb. per cu. ft., $f_c = 250$ lb. per sq. in., and $\phi_0 = 30$ degrees.

[25] Engr., State Div. of Water Resources, Sacramento, Calif.

[25a] Received by the Secretary, July 6, 1931.

With these data and notation in Fig. 34: $\phi_k = 34.71512°$, $a = 2.15286°$, and $\frac{h}{h_v} = 0.97564$.

Furthermore, let $\frac{v - v_1}{2} = q$; then,

$$t = \frac{\gamma_w b h e^{2q} \sec^2 \phi_k}{f_c [\tan \phi'_0 \tan \phi' + 1]} \quad \ldots\ldots (55)$$

$$x' = v \tan \phi'_0 + c (\phi' - \phi_k) \quad \ldots\ldots (56)$$

and,

$$W_c = \gamma_w b h dv [\tan \phi' - \tan \phi_k] \quad \ldots\ldots (57)$$

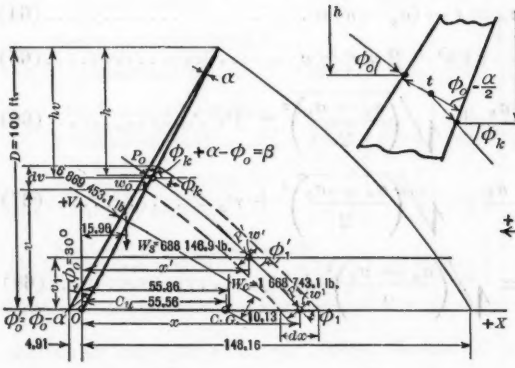


FIG. 34.

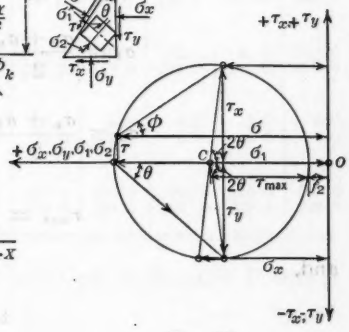


FIG. 35.

Taking moments about Point O (Fig. 34):

$$M_{co} = \gamma_w b h d v [v \tan \phi'_0 (\tan \phi - \tan \phi_k) + c \tan \phi' (\phi' - \phi_k) - (v - v_1)] \quad \ldots\ldots (58)$$

and,

$$\tan \phi' = \sqrt{e^{2q} \sec^2 \phi_k - 1}$$

In case $v_1 = 0$, then $\phi' = \phi_1$.

The stresses as computed by the catenary method are:

Vertical normal stress:

$$\sigma_y = f_c \sin^2 \phi' \quad \ldots\ldots (59)$$

Horizontal normal stress:

$$\sigma_x = f_c \cos^2 \phi' \quad \ldots\ldots (60)$$

and,

Shearing stress:

$$\sigma_x = f_c \sin \phi' \cos \phi' \quad \ldots\ldots (61)$$

Due to the fact that the face of the buttress makes an angle, $\beta = \phi_E - \phi_0 + \alpha = 6.86845°$, with a plane normal to the catenary at that point there is very slight shearing stress at the face, $\tau = f_c \sin \beta \cos \beta = 29.7$ lb. per sq. in., the compressive normal stress being $\sigma = f_c \cos^2 \beta = 246.4$ lb. per sq. in.

Vertical normal stresses by the ordinary method were computed by the formula:

$$\sigma_y = \frac{\Sigma V}{A} \pm \frac{\Sigma M}{S} \quad \ldots\ldots (62)$$

The stresses at the up-stream face are:

$$\sigma_y = \frac{\Sigma V}{A} - \frac{\Sigma M}{S} \quad \ldots\ldots (63)$$

$$\tau_x = \tau - (\sigma_y - \sigma)\, n \quad \ldots\ldots (64)$$

$$\sigma_x = (\sigma_y - \sigma)\, n^2 - 2\, n\, \tau + \sigma \quad \ldots\ldots (65)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (66)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (67)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (68)$$

and,

$$\tan \theta = \frac{\tau_y}{\sigma_1 - \sigma_y} \quad \ldots\ldots (69)$$

in which, $n = \tan \phi'_0 = 0.52828$.

The stresses at the down-stream face are:

$$\sigma_y = \frac{\Sigma V}{A} + \frac{\Sigma M}{S} \quad \ldots\ldots (70)$$

$$\tau_x = m\, \sigma_y \quad \ldots\ldots (71)$$

$$\sigma_x = m\, \tau_x \quad \ldots\ldots (72)$$

$$\sigma_1 = \sigma_y (1 - m^2) \quad \ldots\ldots (73)$$

$$\sigma_2 = 0 \quad \ldots\ldots (74)$$

and,

$$\tau_{max} = \frac{1}{2} \sigma_1$$

in which, $m = \cot \phi'$.

Analytical stress computations were checked by Mohr's circle as adapted by the writer to the general condition of stress as shown on Fig. 35. Table 1 shows that the two methods give essentially the same results, at least for the

TABLE 1.—COMPARISON OF STRESSES COMPUTED BY ORDINARY METHOD AND BY CATENARY METHOD

(Pounds per square inch)

Item No. (1)	Kind of stress (2)	REFERENCE POINTS ON THE DAM (SEE FIGS. 36 AND 37) A (3)	a (4)	B (5)	b (6)	C (7)	c (8)	D (9)	d (10)
	Vertical Normal Stress, σ_y:								
1	Catenary method	81.1	174.1	81.1	153.5	81.1	136.8	81.1	117.1
2	Ordinary method	79.3	178.7	78.9	155.8	78.3	139.1	77.7	123.8
	Horizontal Shearing Stress, τ_x:								
3	Catenary method	117.0	115.0	117.0	121.7	117.0	124.5	117.0	124.8
4	Ordinary method	118.0	118.0	118.3	123.5	118.5	126.5	118.8	131.9
	Horizontal Normal Stress, σ_x:								
5	Catenary method	168.9	75.9	168.9	96.5	168.9	113.2	168.9	132.9
6	Ordinary method	168.4	78.0	168.2	97.9	168.1	115.1	168.0	140.5
	First Principal Stress, σ_1:								
7	Catenary method	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
8	Ordinary method	250.0	256.6	250.0	253.7	250.0	254.2	250.0	268.3
	Second Principal Stress, σ_2:								
9	Catenary method	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	Ordinary method	—2.2	0.0	—2.9	0.0	—3.5	0.0	—4.3	0.0
	Maximum Shear Stress, τ_{max}:								
11	Catenary method	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
12	Ordinary method	126.1	128.3	126.5	126.8	126.7	127.1	127.1	132.1

particular example selected. Figs. 36 and 37 will serve to identify Points *A* to *D* and *a* to *d* in this table.

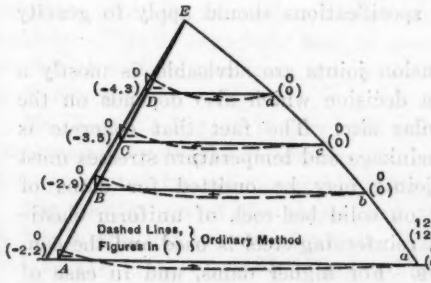


FIG. 36.—DIAGRAM OF SECOND PRINCIPAL STRESSES, σ_2.

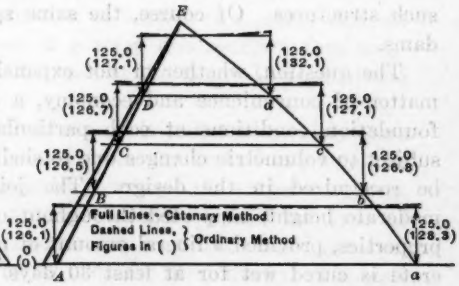


FIG. 37.—DIAGRAM OF MAXIMUM PRINCIPAL STRESSES, τ_{max}.

In conclusion, the writer wishes to mention that joints should not enter the problem until shrinkage and other structural items have been discussed because if the buttress is treated as an inelastic body the separate catenary columns will be in equilibrium regardless of whether or not they are joined together.

HERMAN SCHORER,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—Mr. Girand properly calls attention to the factor of safety against overturning and suggests that two-thirds of the actual weight of the concrete be used for design purposes. This assumption evidently is identical in effect with that of using a normal unit weight for concrete, but a unit weight of water and a design stress, which are both 50% larger than the usual values.

Since the unit weight of concrete is not subject to changes, the latter assumptions would perhaps be somewhat more logical, because the unit weight of the impounded fluid may actually increase, due to gradual or sudden silting of the reservoir, depending on the character of the drainage area. According to Charles Terzaghi,[27] M. Am. Soc. C. E., the consolidation of mud deposits is an extremely slow process, so that the full fluid pressure may be exerted over a considerable period of time.

The French Commission, which reported on the third failure of the La Habra Dam,[28] which was of the gravity type, has estimated a fluid weight of 81 lb. per cu. ft. for the impounded mud and water caused by a sudden torrential flood.

Under such conditions, a higher compressive stress would be entirely justified on account of the increase in the compressive strength of concrete with age. It is much more important to avoid tensile stresses in the concrete and sliding of the buttresses under such extreme, but not impossible, load conditions. As pointed out by Mr. Girand, this large factor of safety can be obtained with comparatively small additional quantities, probably not exceeding 10% in most cases.

A conservative building code for dams, specifying a fluid weight of 90 lb. per cu. ft., a weight of concrete of 140 lb. per cu. ft., a principal compressive stress of 600 lb. per sq. in., no tension, and a maximum sliding factor of 0.70, would lead to an adequate design, consistent with the importance of such structures. Of course, the same specifications should apply to gravity dams.

The question, whether or not expansion joints are advisable, is mostly a matter of convenience and economy, a decision which also depends on the foundation conditions at each particular site. The fact that concrete is subject to volumetric changes due to shrinkage and temperature stresses must be recognized in the design. The joints may be omitted for dams of moderate height, supported throughout on solid bed-rock of uniform elastic properties, provided a liberal amount of reinforcing steel is used and the concrete is cured wet for at least 30 days. For higher dams, and in case of non-uniform foundation conditions, the provision of permanent joints has decided advantages, because large masses of concrete are thereby subdivided into convenient construction units, and the amount of reinforcing steel may be considerably reduced.

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[26] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

[26a] Received by the Secretary, June 23, 1931.

[27] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 278.

[28] "La Rupture du Barrage de L'Habra," *Le Génie Civil*, March 24, 1928, p. 283.

Mr. Girand suggests locating expansion joints, if any, in the direction of the second principal stress. Evidently, a joint so placed could not act as intended, because the first principal stress would prevent movement on account of friction. His proposed method of placing concrete would certainly be the ideal way. In case of a monolithic buttress most of the reinforcing steel should be placed in the direction of the second principal stress, as pointed out by Mr. Girand in Fig. 14.

Mr. Noetzli has evidently worked along similar lines in the design and, therefore, has come to fairly identical conclusions. The writer is aware of the fact that inclined joints have been used in several dams before this paper was published; but not, however, before the time of his original conception and formal presentation. This feature is inherent with the principle of self-supporting arched column units, where the columns may be built integral or adjacent to each other, and therefore, the writer does not hesitate to claim priority in this matter.

The joints shown by Mr. Noetzli in Fig. 15 and Fig. 16 stop a short distance below the deck, causing a local stress concentration, which could be eliminated by extending the joint through the deck. Mr. Noetzli mentions that in some cases the stress analysis indicated tension in the direction of the second principal stress which undoubtedly made it necessary to stop the joint below the deck and to provide extra steel reinforcement in the deck zone.

It is not sufficient that only the shearing stresses be zero along the plane of the joint; it is still more important that the normal stresses (second principal stress) should not be tension. Steel reinforcement cannot generally be relied upon to transmit tensile stresses to the foundation and a slight amount of compression in the direction of the second principal stress is, therefore, desirable.

With the ordinary buttress shape of rectangular horizontal cross-section and other arbitrary dimensions the elimination of tension is a rather difficult problem. This is especially true in case of a steep down-stream face combined with a steep up-stream face, as in the case of the Gleno Dam, analyzed by A. Stucky.[29]

Mr. Noetzli's Fig. 16 is very instructive because it indicates a fairly close agreement between the σ_x stresses, as derived from the arched columns and those computed for the monolithic buttress by means of the trapezoidal rule.

His comparison of the round-head buttress with the cantilever type of deck is interesting, but not convincing. The stresses in a disk-shaped member, such as Fig. 17(*a*), cannot be computed by the ordinary beam theory. The tests by E. Moersch[30] have shown conclusively the beneficial influence of the haunches on the reduction of the shearing stresses. For a more exact analysis, supported by photo-elastic stress measurements, the reader is referred to the work of Akira Miura.[31]

[29] "Der Talsperrenbruch im Val Gleno," by A. Stucky, *Schweizerische Bauzeitung*, Vol. 83 (1924), p. 74.

[30] "Der Eisenbetonbau," by E. Moersch, Vol. I, Second Half, Fifth Edition, pub. by Konrad Wittwer, Stuttgart, 1922, p. 197.

[31] "Spannungskurven in rechteckigen und keilförmigen Trägern," by Akira Miura, pub. by Julius Springer, Berlin, 1928.

The rounding of the up-stream face may have a favorable influence on the stress distribution, but the total saving in concrete, if any, will be small, because the yardage in the deck is a small percentage of the total yardage in the dam. Since the deck is a very vital part of the structure the writer would not advocate the omission of reinforcing steel in a deck of this type. It should be pointed out that the dead load stresses are not radial and the possibility of shrinkage and temperature cracks, as well as earthquake stresses, may lead to serious consequences in the case of a plain concrete deck. Since reinforcing steel should be provided in either case, the advantages of a straight cantilever deck in the ease of placing steel and form work are evident.

On account of his pioneer work in the analysis of principal stresses in multiple arch dams,[32] Mr. Jakobsen is well qualified to appreciate the possible advantages of a direct design method. The result of his comparison in a practical case is gratifying and demonstrates the usefulness of the proposed principle.

As pointed out by Mr. Jakobsen, the arrangement shown in Fig. 5 introduces the possibility of an upward thrust on the deck. This uplift force should be combined with the other forces acting on the monolithic column head, as mentioned by the writer under the heading, "Design Example."

The writer thoroughly agrees with Mr. Jakobsen that inclined joints, if used at all, should be continuous through deck and buttress down to the foundation.

Mr. Davis prefers the use of a more laborious method, because in his opinion the results obtained by considering the buttress as a monolith will be less favorable than those obtained from the independent arch columns. If the adjacent arched columns were not designed for the same compressive stress throughout, then, the monolithic and independent stress system could not be identical. By computing an actual design example, Mr. Davis will convince himself that the results are almost identical with those obtained by the trapezoidal rule. In order to be consistent, the same assumption regarding the action between deck and buttress, either according to Fig. 4 or Fig. 5, must be made in both stress computations.

Mr. Davis correctly states that the method can only be applied for "reservoir full." For lower reservoir levels the principal stress system must necessarily be different. Perhaps he overlooks the fact that the proposed principle is not a stress analysis method, but only a means to dimension the structure directly for the most dangerous case of loading under given design assumptions. If several possible load conditions could produce maximum or minimum stresses of the same order, then, such a direct method could not give the desired result.

In the design of dams there are generally only two important cases of loading, namely "reservoir full" and "reservoir empty." In gravity dams, both these load conditions produce maximum and minimum stresses of the same order. In arch dams and buttressed dams, however, the most dangerous load condition is found with reservoir full, so that a structure dimensioned

[32] "Stresses in Multiple-Arch Dams," by B. F. Jakobsen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

for this case will generally be safe under all other load conditions. In no case, however, should the designer neglect to investigate other conditions of loading, even if they are apparently safe. The analysis by means of the trapezoidal rule will be sufficient for such cases.

Mr. Eremin points out the possibility of secondary deck stresses due to unequal column deformations. If the deck is separated from the monolithic buttress by means of a continuous joint, as shown in Fig. 4, then the buttress can only deform as a whole. Since the deck is comparatively much more flexible than the stiff buttress, it is evident that the secondary deck stresses are small, but their magnitude may be determined from the buttress deformation.

The writer agrees with Mr. Eremin that the arrangement of Fig. 4 (independent deck) should not be combined with independent column units because the relative deformation of the individual column units may introduce high local deck stresses near the column corners. With the arrangement of Fig. 5 the possible interaction between individual deck strips can practically be eliminated by providing a plastic filler in the deck joint area.

Mr. Eremin points out that the foundation deformations should be considered in an exact design of the buttress. These deformations may be estimated according to the methods proposed[33] by Fredrik Vogt, Assoc. M. Am. Soc. C. E.

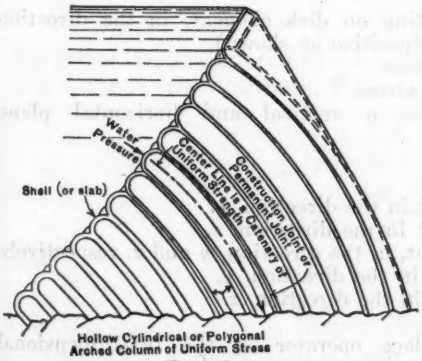

Water Pressure
Shell (or slab)
Center Line is a Catenary of Uniform Strength
Construction Joint or Permanent Joint
Hollow Cylindrical or Polygonal Arched Column of Uniform Stress

FIG. 38.

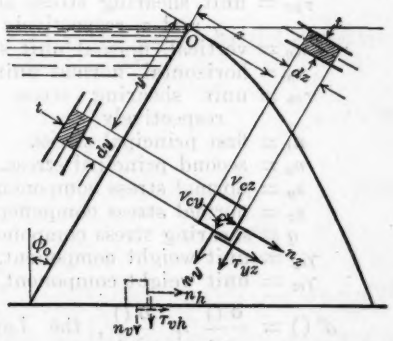


FIG. 39.

Mr. Eremin's suggestion to increase the possible height of Dr. Rossin's buttress type by the use of hollow columns, is interesting. Fig. 38 shows the adaptation of the arched column principle to a tenatative design with reinforced hollow column sections. The construction of Dr. Rossin's type, or a type similar to that of Fig. 38, is economical in locations where cheap skilled labor is available in connection with comparatively expensive material.

Professor Kalman's able discussion is based mostly on theoretical considerations and, therefore, can be answered only on that basis. For this purpose it will be necessary to introduce the fundamental relations of the theory of elasticity, as applied to structures of this type, with the usual assumptions made for the elastic properties of the material.

[33] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 554.

Under the heading "Comparison with the Principle of Linear Stress Distribution," Professor Kalman states:

"Should the buttress be treated as a column when it is monolithic? Decidedly, it is not a column. Should it be treated as a beam? If so, the trapezoidal rule should be applied to the individual sections, and the author is opposed to the application of this rule."

The buttress is neither a column nor a beam, but it is a three-dimensional elastic body. Since one dimension is small compared to the other dimensions it may be defined as a disk of variable thickness, symmetrical about its middle plane. This disk, in contrast to a plate or slab, is assumed to be loaded by forces acting in the direction of the middle plane only. If the thickness is comparatively small, then the stresses in the direction normal to the middle plane can be neglected, resulting in a two-dimensional, or plane (not necessarily straight-line) stress system, as defined by Föppl.[34]

With reference to Fig. 39, the following definitions are introduced:

O = origin of co-ordinate system.
y, z = co-ordinates of a small element, dy, dz.
t = thickness of buttress, given as a function of y and z.
n_y = normal unit stress on small element, acting in the direction, y (tension positive).
n_z = normal unit stress on small element, acting in the direction, z (tension positive).
τ_{yz} = unit shearing stress acting on disk element, in the direction, y and z, respectively (positive as shown).
n_v = vertical, normal unit stress.
n_h = horizontal, normal unit stress.
τ_{vh} = unit shearing stress in a vertical and horizontal plane, respectively.
n_1 = first principal stress.
n_2 = second principal stress.
s_y = normal stress component in the direction, y.
s_z = normal stress component in the direction, z.
q = shearing stress component, in the direction, y and z, respectively.
γ_{cy} = unit weight component, in the direction, y.
γ_{cz} = unit weight component, in the direction, z.

$\Delta^2 () = \frac{\partial ()}{\partial y^2} + \frac{\partial ()}{\partial z^2}$, the Laplace operator in the two-dimensional system, y, z.

For the purpose of the following analysis it will be most convenient to consider the stress components, instead of the unit stresses, acting on a small element. The stress components are:

$$s_y = t\, n_y \quad \ldots\ldots (75)$$

$$s_z = t\, n_z \quad \ldots\ldots (76)$$

$$q = t\, \tau_{yz} \quad \ldots\ldots (77)$$

[34] "Drang und Zwang," by August Föppl and Ludwig Föppl, Vol I, Second Edition, pub. by R. Oldenbourg, Munich and Berlin, 1924, p. 244.

In order to maintain equilibrium on a small element the following well-known fundamental equations apply,

$$\frac{\partial s_y}{\partial y} + \frac{\partial q}{\partial z} + t\,\gamma_{cy} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

and,

$$\frac{\partial s_z}{\partial z} + \frac{\partial q}{\partial y} + t\,\gamma_{cz} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

As shown by Föppl,[35] the compatibility between stresses and deformations also requires compliance with the following equation,

$$\nabla^2 (n_y + n_z) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

or, expressed by the stress components,

$$\nabla^2 \left(\frac{s_y}{t} + \frac{s_z}{t} \right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

For a given load system and given buttress dimensions, the three unknowns, s_y, s_z, and q, must be determined by integration of the three simultaneous, differential Equations (78), (79), and (81).

Since t is expressed as a given function of y and z, a function, $T\,(y, z)$, may be determined by partial integration of t with respect to y and z, so that,

$$t = \frac{\partial^2 T}{\partial_y\, \partial_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (82)$$

The stress components can now be expressed as the second partial derivatives of a common stress function, $F\,(y, z)$, defined as follows:

$$s_y = \frac{\partial^2 F}{\partial z^2} - \frac{\partial T}{\partial z}\gamma_{cy} \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

$$s_z = \frac{\partial^2 F}{\partial y^2} - \frac{\partial T}{\partial y}\gamma_{cz} \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

and,

$$q = - \frac{\partial^2 F}{\partial_y\, \partial_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

By substituting Equations (83), (84), and (85) for s_y, s_z, q, and t, in Equations (78) and (79), it may be seen that the equilibrium equations are satisfied.

The compatibility Equation (81) then becomes, by substituting s_y and s_z from Equations (83) and (84),

$$\nabla^2 \left(\frac{1}{t}\frac{\partial^2 F}{\partial y^2} + \frac{1}{t}\frac{\partial^2 F}{\partial z^2} - \frac{1}{t}\frac{\partial T}{\partial y}\gamma_{cz} - \frac{1}{t}\frac{\partial T}{\partial z}\gamma_{cy} \right) = 0 \ldots\ldots (86)$$

Equation (86) represents the general differential equation of the desired stress-function in the most general case of a buttress of variable thickness, subject to any external loads acting in the middle plane, and gravity action.

[35] "Drang und Zwang," by August and Ludwig Föppl, Vol. 1, Second Edition, 1924, pp. 247 and 326.

If the gravity action is neglected, then Equation (86) becomes,

$$\nabla^2 \left(\frac{1}{t} \frac{\partial^2 F}{\partial y^2} + \frac{1}{t} \frac{\partial^2 F}{\partial z^2} \right) = 0 \quad \ldots\ldots\ldots\ldots\ldots (87)$$

Equation (87) is given by Föppl,[36] but the general integral is not known and, therefore, neither is that of Equation (86). For a disk of constant thickness (t constant), Equation (87) is reduced to the well-known differential equation for Airy's stress function,

$$\nabla^2 \left(\frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} \right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

or, written in the more common form,

$$\frac{\partial^4 F}{\partial y^4} + 2 \frac{\partial^4 F}{\partial y^2 \partial z^2} + \frac{\partial^4 F}{\partial z^4} = 0 \quad \ldots\ldots\ldots\ldots\ldots (89)$$

The general integral of Equation (89) is a complex function, as given by Föppl,[37] but such a solution only applies to a buttress of constant thickness, or a slice of a gravity dam section, contained between two parallel vertical planes, normal to the up-stream face.

Any exact solution of the most general case must be based on Equation (86). For a given load system and given buttress dimensions there can exist only one stress function, which satisfies this differential equation and such a stress function then defines the exact stress system.

As pointed out by Professor Kalman, the proposed method of dimensioning a buttress of uniform strength results in a deviation from the trapezoidal rule, especially in the case of a vertical up-stream face. The following analysis, therefore, will be derived for a vertical up-stream face, in which case the maximum discrepancy from the exact solution can be expected. With $\phi_0 = 0$, and the designations of Fig. 39, from Equations (2), (4), and (10), in the case of a weightless deck, the thickness of the buttress at any point is defined by the equation,

$$t = \frac{b \gamma_w}{c \gamma_c} \left(y + c \log \cos \frac{z}{c} \right) \sec^2 \frac{z}{c} \quad \ldots\ldots\ldots\ldots\ldots (90)$$

The function, T, defined by Equation (82), is found by partial integration of Equation (90) with respect to y and z, which gives,

$$T = \frac{b c \gamma_w}{\gamma_c} \left[\tan \frac{z}{c} \left(1 + \frac{y}{2c} - \log \cos \frac{z}{c} \right) - \frac{z}{c} \right] y \quad \ldots\ldots (91)$$

For $\phi_0 = 0$, $\gamma_{cy} = \gamma_c$ and $\gamma_{cx} = 0$, therefore,

$$\frac{\partial T}{\partial y} \gamma_{cz} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (92)$$

and,

$$\frac{\partial T}{\partial z} \gamma_{cy} = b \gamma_w \left(\frac{y}{2c} + \log \cos \frac{z}{c} \right) y \sec^2 \frac{z}{c} \quad \ldots\ldots\ldots\ldots (93)$$

[36] "Drang und Zwang," Vol. I, Second Edition, 1924, p. 326.
[37] *Loc. cit.*, p. 252.

The desired stress function, F, as given by the following equation has been found by trial:

$$F = -b\gamma_w\left[\frac{y^3}{6} + \frac{c\,y}{2}\log\cos\frac{z}{c}\left(y + c\log\cos\frac{z}{c}\right) + c\int\int\tan^2\frac{z}{c}\log\cos\frac{z}{c}\,dz\,dz\right] \quad \ldots\ldots(94)$$

With reference to Equations (83), (84), (85), (92), and (93), the following values of the stress components are obtained from Equation (94),

$$s_y = -b\,\gamma_w\left(y + c\log\cos\frac{z}{c}\right)\tan^2\frac{z}{c} \quad \ldots\ldots(95)$$

$$s_z = -b\,\gamma_w\left(y + c\log\cos\frac{z}{c}\right) \quad \ldots\ldots(96)$$

and,

$$q = -b\,\gamma_w\left(y + c\log\cos\frac{z}{c}\right)\tan\frac{z}{c} \quad \ldots\ldots(97)$$

At the up-stream face, with $z = 0$, the stress components give the following values,

$$s_y = 0 \quad \ldots\ldots(98)$$

$$s_z = -b\,\gamma_w y \quad \ldots\ldots(99)$$

$$q = 0 \quad \ldots\ldots(100)$$

These boundary conditions evidently correspond to a buttress of unlimited height, with the water level at the origin, O, of the co-ordinate system, as shown in Fig. 39, and a weightless deck of the width, b.

With reference to Equations (75), (76), (77), (90), and c from Equation (1), the unit stresses at any point of the buttress become,

$$n_y = \frac{s_y}{t} = -f_c\sin^2\frac{z}{c} \quad \ldots\ldots(101)$$

$$n_z = \frac{s_z}{t} = -f_c\cos^2\frac{z}{c} \quad \ldots\ldots(102)$$

and,

$$\tau_{yz} = \frac{q}{t} = -f_c\sin\frac{z}{c}\cos\frac{z}{c} \quad \ldots\ldots(103)$$

The sum of the normal stresses at any point becomes, therefore,

$$n_y + n_z = -f_c\left(\sin^2\frac{z}{c} + \cos^2\frac{z}{c}\right) = -f_c \quad \ldots\ldots(104)$$

Substituting Equation (104) in the compatibility Equation (80), and considering that f_c is a constant,

$$\nabla^2(n_y + n_z) = \frac{\partial^2(-f_c)}{\partial y^2} + \frac{\partial^2(-f_c)}{\partial z^2} = 0 \quad \ldots\ldots(105)$$

Since the stress function, Equation (94), complies with the compatibility Equation (80), it also satisfies the differential Equation (86), as well as the

equilibrium Equations (78) and (79) and, therefore, defines the actual stress system for the given buttress shape and the aforementioned boundary conditions.

From Equations (101), (102), and (103), the principal stresses, n_1 and n_2, at any point, are found from the well-known equations;

$$n_1 = \frac{n_y + n_z}{2} - \frac{1}{2}\sqrt{4\,\tau\,\frac{2}{y\,z} + (n_y - n_z)^2} = -f_c \quad \ldots\ldots(106)$$

and,

$$n_2 = \frac{n_y + n_z}{2} + \frac{1}{2}\sqrt{4\,\tau\,\frac{2}{y\,z} + (n_y - n_z)^2} = 0 \quad \ldots\ldots(107)$$

Designating the direction of the first principal stress by the angle, β, with the direction, z, then,

$$\beta = \frac{1}{2}\tan^{-1}\frac{2\,\tau_{yz}}{n_z - n_y} = \frac{z}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots(108)$$

For a vertical up-stream face, from Equation (4),

$$\frac{z}{c} = \phi_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(109)$$

Equation (109) indicates that the direction of the first principal stress at any point coincides with the direction of the tangent of the catenary of uniform strength through that point. Equation (106) shows that the first principal stress at any point is constant, compression, and equal to the design stress, f_c. Equation (107) shows that the second principal stress is zero at any point.

From the foregoing derivations it must be concluded, therefore, that the buttress dimensioned according to Equation (90) actually gives the exact solution for the buttress of uniform strength in the case of unlimited height, vertical up-stream face, weightless deck, and full reservoir.

A similar solution also can be derived in case of a buttress of unlimited height, the up-stream face of which is inclined at the angle, ϕ_0, to the vertical, and the thickness at any point defined by the equation,

$$t = \frac{b\,\gamma_w \cos\phi_0\, y}{f_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(110)$$

The shape of such a buttress is similar to that of the upper part of Fig. 2, consisting of a triangular up-stream face, triangular horizontal cross-sections, and a straight down-stream face, normal to the plane of the up-stream face. The proposed method gives such a shape in case the unit weight of the concrete is assumed to be zero and the corresponding stress function is,

$$F = -b\,\gamma_w \cos\phi_0 \frac{y^3}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(111)$$

The unit stresses, by similar derivations as the preceding, are:

$$n_y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(112)$$

$$n_z = -f_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(113)$$

$$\tau_{yz} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(114)$$

Since the shearing stress is zero, the co-ordinate system, in this case, also defines the principal stress system, the first principal stress being constant at any point, and again equal to the design stress, f_c, the second principal stress being zero. Such a buttress, therefore, can be conceived as built up of straight, weightless columns, the axes of which are normal to the up-stream face. The vertical stresses in a horizontal section are also uniform and, in this case only, are identical with those obtained from the trapezoidal rule.

Since the proposed design principle represents the exact solution for the two most extreme cases it is logical to conclude that the same theory also applies to intermediate cases and, therefore, Professor Kalman's sweeping statement, to the effect that there is absolutely no proof of its validity, must be completely rejected.

With the designations of Fig. 39, the stresses in a horizontal section, from Equations (98), (99), (103), and (109), become,

$$n_v = -f_c \sin^2 \phi_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (115)$$

$$n_h = -f_c \cos^2 \phi_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (116)$$

and,

$$\tau_{vh} = -f_c \sin \phi_1 \cos \phi_1 \ldots\ldots\ldots\ldots\ldots\ldots (117)$$

Equations (115) and (117) are identical with Professor Kalman's Equations (49) and (50). However, the writer is unable to check Professor Kalman's stress diagrams, Fig. 29 (*b*) and Fig. 30 (*b*), because the angle, ϕ_1, (see Fig. 1), can never assume a value of 90° and for this reason the shearing stresses, as computed from Equation (50), or Equation (117), can never become zero at the down-stream face.

Professor Kalman's general conclusions, based on these diagrams, are therefore misleading, especially also due to the fact that the assumptions of Figs. 29 and 30 are not clearly stated.

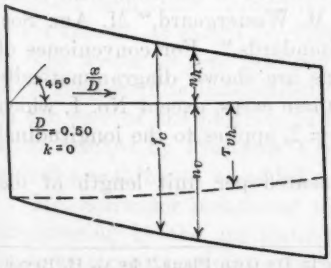


FIG. 40.

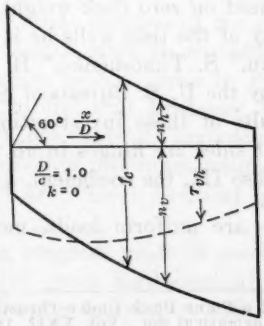


FIG. 41.

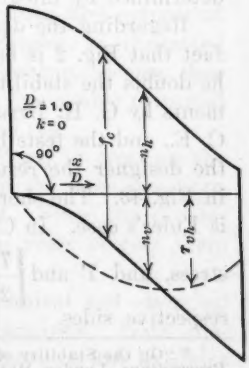


FIG. 42.

Fig. 40 shows the normal and shearing stresses in a horizontal section, based on a buttress with $\frac{D}{c} = 0.50$, $\phi_0 = 45°$, and $k = 0$. These conditions, except for the weightless deck, are typical within the ordinary practical

range. The stress diagram indicates that, in this case, the assumption of the trapezoidal rule for the n_v stresses would give a satisfactory agreement with the actual stress conditions.

Fig. 41 is based on a buttress with $\frac{D}{c} = 1.0$, $\phi_o = 30°$, and $k = 0$, and represents approximately the conditions at the upper practical range, because an even steeper up-stream slope would generally give sliding factors that are too high. It will be noted that the deviation from the trapezoidal has increased. Although the discrepancy is not large, the shape of the n_v-stress diagram indicates that the trapezoidal rule would not necessarily give stresses on the safe side.

Fig. 42 illustrates the conditions in the case of a vertical up-stream face which is only of theoretical interest due to excessively high sliding factors combined with a waste of material. The trapezoidal rule would indicate a triangular n_v-stress diagram, which, in this case, is a rather poor approximation to the actual stress conditions.

Of course, the foregoing comparisons with the trapezoidal rule only apply to a buttress of uniform strength and do not permit any general conclusions for other buttress shapes, especially if such shapes deviate greatly from those obtained with the proposed design method.

However, these typical examples sufficiently demonstrate that in a reasonably economical design a fairly close agreement with the actual conditions can be expected from a principal stress system derived by means of the trapezoidal rule, provided the up-stream slope is not too steep. The satisfactory results obtained by Mr. Noetzli and Mr. Jakobsen seem to support this point. There is absolutely no objection to a comparison of the findings in this manner as long as the designer keeps in mind the fact that the trapezoidal rule is an assumption and that the actual stress system can only be determined by integration of the differential Equation (86).

Regarding the design example, Professor Kalman evidently overlooks the fact that Fig. 2 is based on zero deck weight, whereas Fig. 11 is not. Since he doubts the stability of the thin walls he is referred to the analytical treatments by G. H. Bryan,[38] S. Timoshenko,[39] H. M. Westergaard,[40] M. Am. Soc. C. E., and the tests by the U. S. Bureau of Standards.[41] For convenience of the designer the results of these investigations are shown diagrammatically in Fig. 43. The short sides are hinges in all these cases, except No. I, which is Euler's case. In Case III, the coefficient, $k = 2$, applies to the longitudinal stress, and T and $\frac{T}{2}$ are uniform loads, measured per unit length of the respective sides.

[38] "On the Stability of a Plane Plate Under Thrusts in Its Own Plane," by G. H. Bryan, *Proceedings*, London Mathematical Soc., Vol. XXII, 1890, p. 54.

[39] "Einige Stabilitätsprobleme der Elastizitätstheorie," by S. Timoshenko, *Zeitschrift für Mathematik und Physik*, Vol. 58, 1909-10, p. 337; and "Applied Elasticity," by S. Timoshenko and T. M. Lessels, Westinghouse Technical Night School Press, Pittsburgh, Pa., 1925, p. 291.

[40] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

[41] "Compressive Strength of Column Web Plates and Wide Web Columns," *Technologic Papers No. 327*, U. S. Bureau of Standards, Washington, D. C., Govt. Printing Office, 1926.

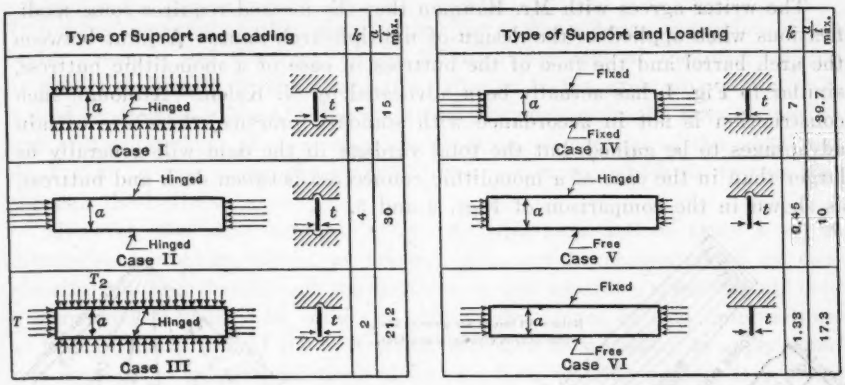


FIG. 43.—BUCKLING STRESS OF THIN PLATES: $F_{cr} = k \frac{n^2 E}{12} \left(\frac{t}{d}\right)^2$

Mr. Floris doubts that Castigliano's theorem can be applied to the stress distribution in gravity dam profiles and still less to a buttress type, as suggested in this paper, which is three-dimensional. Föppl[42] has shown conclusively also that in a three-dimensional elastic body the actual stress system will produce the least amount of work. In fact, every elastic body has, of course, three dimensions and the definition of one-, or two-dimensional stress systems only serves to introduce permissible simplifications in the analysis when certain dimensions are comparatively small.

Since the general Equation (86) defines the actual stress system, it also conforms to the principle of least work and so do Equations (94) and (111) in the special case of a buttress of uniform strength.

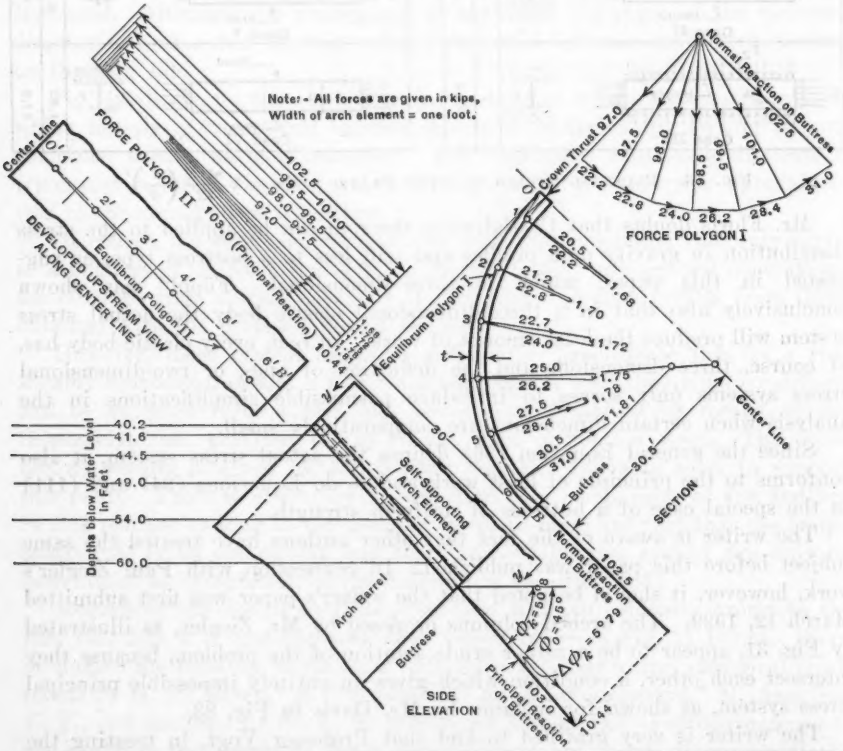
The writer is aware of the fact that other authors have treated the same subject before this paper was published. In connection with Paul Ziegler's work, however, it should be noted that the writer's paper was first submitted March 12, 1929. The arched columns proposed by Mr. Ziegler, as illustrated by Fig. 31, appear to be a rather crude solution of the problem, because they intersect each other, a condition which gives an entirely impossible principal stress system, as shown for instance by Mr. Davis in Fig. 23.

The writer is very gratified to find that Professor Vogt, in treating the same subject quite independently, has evidently arrived at the same equations in the case of the buttress of absolute minimum weight. Since the conclusions reached by Professor Vogt are almost identical in every respect there remains nothing to be added.

Mr. Schwager points out the progress made in mechanical and electrical engineering by the application of exact mathematical analysis. With concrete as a construction material and a foundation with more or less irregular elastic properties even a theoretically correct solution becomes necessarily an approximation. However, the stress system obtained from exact analytical methods under ideal elastic conditions must always serve as the most reliable guide in a practical design.

[42] "Drang und Kwang," by August Föppl and Ludwig Föppl, Vol. I, Second Edition, 1924, p. 99.

The writer agrees with Mr. Bauman that the method requires some modifications when applied to the design of multiple-arch dams. A joint between the arch barrel and the face of the buttress in case of a monolithic buttress, similar to Fig. 4, has actually been advocated by N. Kelen.[43] Although such construction is not in accordance with standard practice, there are certain advantages to be gained, but the total yardage in the dam will generally be larger than in the case of a monolithic connection between deck and buttress, as shown in the comparison of Figs. 4 and 5.



Arch Voussoir	Upstream Length ft.	Water Depth ft.	Unit Weight lb. per cu. ft.	Water Pressure lb.	Mean Length ft.	Mean Thickness ft.	Unit Weight lb. per cu. ft.	Weight of Voussoir lb.	Component normal and parallel to Arch Axis
1	8.15	40.2	62.5	20 500	7.9	2.01	150	2 380	1 680
2		41.6		21 200		2.03		2 410	1 700
3		44.5		22 700		2.05		2 430	1 720
4		49.0		25 000		2.08		2 470	1 750
5		54.0		27 500		2.12		2 520	1 780
6		60.0		30 500		2.18		2 580	1 810

FIG. 44.

In order to apply the proposed method to a monolithically connected arch barrel it will be consistent to consider self-supporting arch elements of uniform strength, which are curved in two directions, as shown in Fig. 44.

[43] "Die Staumauern," by N. Kelen, pub. by Julius Springer, Berlin, 1926, p. 171.

It follows then that in the case of long span arches a large proportion of the water pressure and the arch weight in the lower part of the deck will be transmitted directly to the foundation, similar to the arch ring action suggested by Professor Vogt.[44] Nevertheless, the yardage in the buttress proper will still exceed that of short span construction, on account of the additional dead load in the upper parts of the deck structure, which is transmitted through the buttress.

However, the exact solution of the multiple-arch dam is truly a three-dimensional problem, which, at present, can only be approximated by considering the arch barrel and the buttress as two separate, symmetrical, two-dimensional systems. In order to obtain coincidence of the deformations at the up-stream face of the buttress, it will then be necessary to apply equal and opposite, corrective reactions along this face, whereby a secondary stress system can be superimposed to the main stress systems in the arch-end buttress, respectively. The proposed method enables the designer to determine the main buttress deformation due to the shortening effect of the arched columns, as indicated in Fig. 1, Line FT'. In a more exact analysis the foundation deformations should also be taken into consideration in similar manner.

Mr. Bauman's statement that less work is required in shortening the arched columns by bending rather than concentric pressure, is not confirmed by the exact analysis, as derived in answer to Professor Kalman's discussion in case of a monolithic buttress. However, the writer agrees with Mr. Bauman regarding the possibility of eccentric column pressures due to partial loads in case of inclined joint construction. For this reason, a normal joint has been provided near the foundation in Fig. 11, containing a sheet-lead filler to allow slight plastic deformations and a redistribution of secondary stresses.

The elimination of friction in the inclined joints does not present any serious problem. With reservoir full there are no normal forces acting across the joint, and the columns can deform freely. With partial loads and reservoir empty there will be a normal component acting across the joint. However, with the usual up-stream slopes the friction coefficient will seldom be less than 1.0, so that movement could still take place, even with reservoir empty.

Mr. Nelidov contributes a very desirable and instructive comparison of the buttress stresses as given by the proposed catenary method and those obtained by analyzing the same buttress by the ordinary method. According to Mr. Nelidov's Equation (62), the latter method is identical with the trapezoidal rule, commonly assumed to be valid for the vertical normal stress distribution in horizontal sections.

For the buttress investigated, with $\frac{D}{c} = 0.40$, $\phi_0 = 30°$, and $k = 0.10$, the agreement between the two methods is quite satisfactory, whereby the conclusions regarding the possible accuracy of the trapezoidal rule, in answer to

[44] *Transactions*, Am Soc. C. E., Vol. 93 (1929), p. 1288.

Professor Kalman's discussion, are further confirmed by Mr. Nelidov's careful analysis for the given case.

In conclusion, the writer wishes to express his sincere thanks to all those who have so freely discussed this paper. It is gratifying to note that the discussions have been constructive throughout, thereby contributing many important points not treated in the original paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

THE EIGHT-MILE CASCADE TUNNEL GREAT NORTHERN RAILWAY

A SYMPOSIUM

Discussion

BY MESSRS. R. H. KEAYS, AND J. G. SULLIVAN

R. H. KEAYS,[8] M. AM. SOC. C. E. (by letter).[8a]—The writer has been much interested in this Symposium, especially Part III, "Construction Plans and Methods." He only regrets that the description is not much longer, because there are no doubt many important features that might be elaborated.

The railroad authorities are to be congratulated that in the very beginning it was decided to line the Cascade Tunnel completely with concrete and not take chances with a timbered tunnel. Under some circumstances even the initial cost of a concrete lining might be less than that of timber.

The construction of this long railroad tunnel, by the use of a so-called pioneer drift (formerly known as the "two-tunnel system") supplies another example of the application of this method and well illustrates its advantages. Reference has been made to the method in connection with the Simplon, Rogers Pass, and Moffat Tunnels, the Simplon, of course, far antedating that of the other two. As adapted to these tunnels, however, the method shows some variations in detail, and the reasons for its adoption in each case may reveal some important differences.

It will be interesting, even at this late date, to review some of these reasons in the case of the Simplon. Going even farther back in history to the case of the St. Gotthard Tunnel (double track) it will be found that the highest rock temperature recorded was 30.7° cent., under a rock cover of 1 700 m. On the Simplon, the greatest rock cover was 2 200 m. (5 577 ft.) and, calculating from experience at the St. Gotthard, the highest temperature to be expected was 42° cent. On the other hand, to insure efficient service from the workmen in the tunnel, it was considered important to bring the air temperature in the Simplon Tunnel down to 25° cent.

NOTE.—The Symposium on the Eight-Mile Cascade Tunnel, Great Northern Railway, was published in February, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[8] Chf. Engr., Ulen & Co., Athens, Greece.

[8a] Received by the Secretary, April 15, 1931.

At the time the construction of the Simplon was undertaken, it was realized that two tracks were necessary, and it was merely a problem of building one double-track tunnel, or two single-track tunnels. It was not necessary to start the second single-track tunnel, however, until the first was completed if a passing track were provided temporarily in the first tunnel. Therefore, the two-tunnel plan was adopted.

The only method considered to cool the tunnel was by ventilation with large quantities of air. For the supply of fresh air without regard to cooling, the total requirements were calculated at less than 5.0 cu. m. per sec., (176.6 cu. ft. per sec.), but to cool down to 25° cent., it might be necessary to provide for no less than 35 cu. m. per sec. (74 500 cu. ft. per min.).

Computations indicated that the total cost of piping such a quantity of air into the tunnel would be the staggering sum of approximately $7 000 000. Under these circumstances scarcely any calculations were necessary to determine that the best way to provide for the ventilation was to adopt the two-tunnel plan.

The Simplon Tunnel was constructed before the days of modern air drills. A bottom-heading method was used, the heading being drilled with rotary hydraulic drills. The tunnel enlargement drilling was entirely by hand methods. The lining is stone masonry.

As a matter of fact the rock temperatures found were much higher than calculated, amounting as a maximum to 55.4° cent. This, of course, made conditions intolerable, and it was necessary to supplement the cooling effect of the air by the use of large quantities of cooling water. An insulated water main, 25.3 cm. (10 in.) in diameter, was installed therefore in the pioneer tunnel. Through this pipe was pumped a maximum of 79 liters per sec. (2.8 cu. ft. per sec.). In addition to this pipe, there were two smaller insulated water mains for the conveyance of high-pressure water to the headings for use in the drills.

The water from the large pipe was used mainly to supply sprinkling nozzles installed in the main tunnel, between the last cross-cut and the enlargement operations. By far, the greatest number of men were employed in the enlargement, and it was necessary to protect them. These nozzles blocked the transportation to the heading of the main tunnel, so that it was necessary for all heading transportation to cross over to the pioneer tunnel at the cross-cut nearest the heading. The maximum water temperature at the nozzles was 17° cent.

Some of the cold water, however, was conducted to the headings and was used for sprinkling the walls and wetting down the outside of the ventilating pipes leading to the headings. The ventilation layout for the headings was similar to that described by Mr. Baxter for the Cascade Tunnel.

The foregoing information in regard to the Simplon Tunnel has been discussed in an interesting manner by Professor C. Andreae.[9]

[9] "Der Baulänger tieflingender Gebirgstunnel," Berlin, 1926.

Before dismissing the subject, the writer wishes to express his curiosity as to the reasons, in the case of the Cascade Tunnel, for estimating rock temperatures as high as 110° Fahr. If the Simplon Tunnel had any influence in this decision, it should be remarked that as far as rock temperatures are concerned Alpine tunnels seem to be in a class by themselves and of these, the Simplon was much the hottest, even allowing for its greater depth. Rock temperatures in mountain tunnels may have something to do with the age of the mountains, the Alps being of relatively recent origin.

The question of the quantity of air to be provided for ventilating purposes in an ordinary rock heading merits a few remarks. In many cases a large nominal quantity is provided for ventilation, until it is found that, due to light-weight pipe and leaky joints, only a small portion of the air reaches the end of the pipe; and then, of course, the advanced end of the pipe is not at the heading where the men are working, but is usually 100 ft., or more farther back. It is exceptional, therefore, for any of the low-pressure air to reach the heading. On the other hand there is always a supply of high-pressure air at the headings. During drilling operations there can be nothing in the heading but air from the high-pressure mains and during blasting operations the high-pressure mains are usually more or less wide open "to blow out the smoke". The function of the low-pressure air, therefore, is merely to hurry the smoke and bad air out of the tunnel after it has passed the end of the ventilating pipe.

In this connection, the writer knows of one important tunnel, in which a peculiar notion was stoutly maintained, probably by all the tunnel men. It was noted that after blasting (with high-pressure air discharging close to the heading and with the low-pressure system in operation at the same time), there seemed to be a wall of smoke maintained on the heading side of the end of the ventilating pipe and distant therefrom, say, 15 or 20 ft. The idea prevailed, therefore, that the operation of the low-pressure system held the smoke back and would not let it come out. After that the men actually went so far as to stop the ventilator during blasting, until all the smoke had been driven past the end of the low-pressure pipe by the high-pressure air from the heading. This was merely another case of things not always being what they seem. The true explanation is that the jet of air from the ventilator pipe instantly diluted all the smoke in range of the jet in the proportion of, say, 25 to 1, or more, so that the air in this zone seemed quite clear.

A variation of this idea is seen in the case of the tunnel superintendent who always had the high-pressure air line tapped with 1-in. nozzles at regular intervals all the way from the heading to the portal. All these nozzles were carefully pointed toward the portal and, after blasting, they were all opened with the idea of driving the smoke out of the heading. Then, there was the case of the superintendent who insisted on putting on a reverse bend at the heading end of the ventilator pipe, so that the jet would point toward the portal. In this case the only result was that he introduced so much back pressure that he practically stopped the air entirely.

J. G. SULLIVAN,[10] M. AM. SOC. C. E. (by letter).[10a]—Discussion usually implies some difference of opinion, but in the case of this Symposium the writer has only words of praise for the engineers and contractors who so rapidly and successfully carried this work to a satisfactory completion, especially in view of the fact that they had to contend with considerable quantities of water in a tunnel with a gradient entirely in one direction. Beginning in 1892, the writer had charge, as Assistant Engineer, of the construction of that portion of the abandoned line from the west end of the Scenic side-track to a point about half way to Tye. (See Fig. 2.) He has, therefore, taken a somewhat personal interest in the progress and outcome of this project.

It was gratifying to note that the contractors followed the plan of driving a center heading and radial drilling (see Fig. 19). When this plan was adopted for the first time, about 1916, in the construction of the Connaught Double-Track Tunnel on the Canadian Pacific Railway, it was rather severely criticized by some engineers of New York City.[11] Where rock is reasonably stable, this method is much more economical in the construction of a double-track tunnel than it would be in a single-track tunnel.

Temperature.—The actual temperatures of rock in the Connaught Tunnel were practically the same as those shown in Fig. 60, although the distance from the summit of the mountain to the grade of the tunnel was about twice as great as that in the Cascade Tunnel. Before the Connaught Tunnel was started it was predicted by some engineers that serious trouble would be encountered from heat. They went so far as to predict an increase of about 1° Fahr. per 100 ft. of depth.

Double-Track vs. Single-Track.—The writer has the impression that the Great Northern Railway Company might have constructed a double-track tunnel. Using the center-heading and radial-drilling system of construction in rock that is reasonably stable, a double-track tunnel can be constructed at a cost not exceeding 125% of the cost of a single-track tunnel.

Since there is already a pioneer tunnel built for about 68% of the distance and considering that time in the construction of the second tunnel would not be an important factor, the second might be constructed for about 60% of the cost of the first tunnel. If these assumptions are approximately correct, and if the second track would be required within 15 or 20 years, depending on the cost of money, it would have been economical to have constructed a double-track tunnel in the first place; that is, of course, with the understanding that the nature of the rock was such that it would stand without support during construction. In the construction of the Connaught Tunnel it was found that, on account of the increased space, the time required for enlarging and lining would not be appreciably increased.

[10] Pres., Sullivan, Kipp and Chace, Winnipeg, Man., Canada.

[10a] Received by the Secretary, April 17, 1931.

[11] *Engineering News*, January 11, 1917, p. 77.

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DISCUSSIONS

FLOW OF GROUND-WATER AS APPLIED TO DRAINAGE WELLS

Discussion

BY L. STANDISH HALL, ASSOC. M. AM. SOC. C. E.

L. STANDISH HALL,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—This paper is very timely as it offers a theoretical presentation of a subject for which real data are lacking to a large extent. It adds several formulas to the hydraulics of wells. The existing ones on this subject were unsatisfactory for general application since, as pointed out by the author, they covered only one or two special cases. The subject of the paper refers to drainage wells, but the formulas presented are of general application to wells, regardless of their use.

Although the formulas cover quite a range of cases, they are somewhat difficult of application in advance of the drilling and operation of a well, because of the difficulty in determining in advance the conditions which will be met. In fact, several other conditions relative to the flow of ground-water are possible in addition to those presented in the paper. For example, deep wells usually penetrate several water-bearing strata, the transmission coefficients of which are, in many cases, different for each strata penetrated by the well. The Type *A* wells may penetrate several strata with downward percolation to some of the pervious beds and with no downward percolation to the remainder of the beds. A well may also belong to the Type *B* in the upper strata and to the Type *A* in the strata penetrated at greater depth. In pumping wells of this last type, the water level in the well during pumping may be drawn below the bottom of the upper strata, allowing a free discharge into the well from the strata thus exposed.

If a well should penetrate several minor water-bearing strata and be landed as an open-bottom well (Type *C*), a combination of this latter type with either Type *A* or Type *B* is possible. The formula presented by the author for the Type *C* well (Equation (20)), indicating a greater draw-down during pumping than for either the Type *A* or Type *B* well, as shown in Fig. 7, appears to be contrary to general opinion. The writer has made no

NOTE.—The paper by M. R. Lewis, M. Am. Soc. C. E., was published in March, 1931, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[9] Chf. Hydrographer, East Bay Municipal Utility Dist., Oakland, Calif.

[9a] Received by the Secretary, May 1, 1931.

actual measurements of the performance of these wells during pumping but statements of owners indicate that the draw-down is usually small. A well not originally drilled as an open-bottom well may often evolve into this type at points where it penetrates unconsolidated strata, particularly if these strata are not cased off in the well. The writer has in mind an example of a well which was drilled originally to a depth of 194 ft. When it was first pumped, twelve wagon-loads of fine sand were discharged with the water. The present depth of the well is 141 ft., which is evidently the depth at which the main water-bearing strata were encountered. Soundings made with a 6-ft. iron rod indicated that a large cavity existed at this depth. The owner stated that pumping of the well did not lower the water level appreciably. Other similar instances could be cited.

With regard to the application of wells to the drainage of land—while they may furnish a large discharge—deep wells may not accomplish the purpose desired. The lands generally requiring drainage are at a low elevation and frequently receive the drainage from higher lands. If the deep wells penetrate sandy strata interbedded with impervious layers with the dip of the strata following the general ground slope, pumping may drain only strata outcropping on higher ground, particularly if the well is of the *A* type. Under this condition the pumping would lower only the piezometric surface and have very little effect in lowering the water-table in the territory requiring drainage.

Even in cases where the deep wells penetrate pervious beds of sandy strata for their entire depth, they may be disadvantageous for the purposes of draining land. This is particularly true when wells that are being operated are considerable distances apart. In order to lower the water-table between the drainage wells, under these conditions, the water level in the vicinity of the well may have to be lowered excessively. This may result in the necessity of frequent irrigation of the lands near the wells, thereby partly defeating the purposes of drainage.

The shallow well that reaches just below the water-table and is pumped to its utmost capacity (which has been mentioned by the author) appears to be the most efficient for drainage. One difficulty presents itself in the case of the use of wells of this type. The yields of each individual unit may be small, thus requiring that they be spaced closely together. This necessitates many wells and the purchase of much pumping equipment, which in some cases may result in a prohibitive cost.

The use of drainage wells for lowering the water-table in the lower areas of irrigation projects is gaining in favor since it permits of the re-use of the water recovered by drainage, for irrigation of additional acreage. It also makes an irrigation system more flexible by providing an underground supply on which one may draw heavily during years of deficient surface run-off.

The subject presented by the author is one on which additional data need to be collected and available existing data need to be correlated.

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DISCUSSIONS

AN ANALYSIS OF MULTIPLE-SKEW ARCHES ON ELASTIC PIERS

Discussion

BY MESSRS. B. L. WEINER, AND G. D. HOUTMAN

B. L. WEINER,[8] ASSOC. M. AM. SOC. C. E. (by letter).[8a]—Parts I, III, and IV of Professor Rathbun's paper are of as much interest to designers of right arches and other statically indeterminate frames as they are to the designers of skew arches. The author has presented a general method for analyzing indeterminate structures, which has a certain marked advantage over other methods known to the profession.

Generally speaking, a right arch has three unknown reactions for each span, which must be obtained by the solution of simultaneous equations. In a three-span arch, there would be nine unknowns, necessitating nine simultaneous equations, each containing all the unknown quantities. This, in general, would be the case, except for certain simplifications which are possible because of symmetry, or because such conditions as hinged bases make certain unknowns zero, or because of other conditions which assign known values to some of the reactions. For the skew arch, the various standard multiple-span analyses could be applied and Professor Rathbun's theory of single-span arches[9] could be extended to multiple spans. In the skew arch, however, there are six unknowns for each arch, and, in general, eighteen equations with eighteen unknowns in each would be required for the analysis of a three-span skew arch. While this is possible, the mere mechanical difficulty of the process would be great even though the solution of simultaneous equations is no particular hardship if determinates are not used.[10] The difficulty increases with the number of spans, and for n spans, there would be $6n$ equations, each containing $6n$ unknowns.

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in April, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[8] Asst. Engr. and Structural Designer, Westchester County Park Comm., Bronxville, N. Y.

[8a] Received by the Secretary, June 23, 1931.

[9] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[10] See *Proceedings*, Am. Soc. C. E., January, 1931, Papers and Discussions, Table 6, p. 34.

Professor Rathbun has shown a way out, since the method he proposes restricts the number of equations to be solved at any one time to that of one span, that is, in a right arch only three equations with three unknowns, and in a skew arch no more than six equations with six unknowns need be solved at any one time. This is an advantage even though the saving in the numerical work involved may not be very much, since a set of the six equations must be solved for each span and, except for the last span, the equations of one span contain constants which are the unknown reactions of an adjacent span. Furthermore, after the equations for one span are solved, giving the reactions of this span in terms of the reactions of an adjacent span, substitutions must be made to eliminate these reactions which have been obtained from the equations of the adjacent span. Finally, after the equations of the last (end) span have been solved, resulting in numerical values for the reactions of this span, a series of numerical substitutions must be made, working backword from the end span to obtain the reactions of the other spans. Furthermore, since the underlying principle of statically indeterminate analysis is that of deflections, the same deflections which appear as constants must be computed no matter what method is used.

However, even if there is no saving in direct labor, in Professor Rathbun's solution, the process is comparatively easy to follow. In this method, the effect of one span is apparent only in an adjacent span and is not directly carried over into other spans, thus making it possible to break the work up into a series of comparatively short steps. Besides reducing the number of simultaneous equations which have to be solved at any one time, this process has another important advantage.

The writer has found by experience in the drafting-room that even when the analysis of an arch is given in complete algebraic formulas, together with a numerical example, an enormous amount of explanation is required by the computers. Without complete algebraic formulas, the task of getting computers to follow through an analysis is made more difficult. While the standard methods of analysis lend themselves readily to expression in algebraic formulas, it is not generally possible to write one set of equations for an arch of any number of spans. Professor Rathbun's solution makes the process the same for any number of spans. The same procedure is repeated from the left-hand to the right-hand span regardless of the number of intervening spans.

It is unfortunate that the author has not given a set of deflection formulas for the case of abutments hinged at the base. Curiously enough, except for the single span with the footings at the same elevation, the assumption that the bases are hinged makes the theory more difficult, even though there are fewer unknowns. It seems to the writer that, for this case, the multiple-span frame will have to be broken up into fundamental types of structures other than cantilevers, and that a more "elegant" method of obtaining deflections will have to be used instead of that of the single cantilever. The cantilever method of obtaining deflections requires certain corrections when the bases are hinged and at different elevations. These corrections involve considerable laborious work and are difficult to apply.

The case of hinged bases is a very important one. The true state of the footings is that of partial fixity. When the first "Rigid Frames" of one span were designed by the Westchester County Park Commission, the analysis was made for both conditions, that is, with the bases fixed, and with the bases hinged. The sections were proportioned for whichever condition was the more severe. After several designs, it was found possible to design for hinged bases only, as the principal difference was found to be at the bottom of the posts where the dowels provided ample reinforcement for either condition. A similar procedure should be followed in the present case and both conditions taken into account.

In the derivation of the equations for the loading terms, the author has taken into account the effect of forces which are located unsymmetrically with respect to the spandrel walls. The only effect seems to be that the forces, T_x (or t_x), M_x (or m_v), and M_y (or m_u) are changed in the skew arch or introduced in the right arch. Taking the latter case, for simplicity, M_z remains unchanged if the load is shifted from a symmetrical, to an unsymmetrical, position. Since Professor Rathbun distributes the moment uniformly over the width of the section, it follows that as far as bending is concerned, all parts of the section from one spandrel wall to the other are equally affected. The effect of the torsion moment is also symmetrical as far as horizontal shears are concerned, and the vertical shears are also of equal value, but of opposite directions at each face of the bridge. This does not seem possible to the writer, because the load will distribute itself in such a way that the area immediately adjacent to the load will be stressed most severely. The question of load distribution on slabs and arches, which has not yet been solved, enters into this problem.

E. F. Gifford, Assistant Engineer in the Engineering Department of the New York Central Railroad, has made some tests on concrete models of rigid frames. The photographs showing the results and method of testing are reproduced herein through the courtesy of Mr. Gifford. (See Figs. 16 to 25, inclusive.) Fig. 26 is a sketch showing the location of the reinforcement, comprising $\frac{1}{2}$-in. square mesh of No. 14 wire, with two of every three cross-strands cut. The mortar mix was 1 part cement and 2 parts sand. The angle of skew was 45° and the scale of model to prototype was 1:64. The models were tested to destruction and no attempt was made to obtain any quantitative results. The first cracks appeared at the obtuse corners of the top slab and continued approximately perpendicular to the face of the bridge. It is obvious from the photographs, that the unsymmetrical load (Figs. 22, 23, and 25) produces a type of failure different from that of the symmetrical load (Figs. 16, 18, 19, 20, 21, and 24). The photographs seem to show that for the unsymmetrical load there are two types of failures: (1) A local failure in the vicinity of the load due to some still unknown factor; and, (2) a general failure which takes place at points far enough away from the load so as not to be affected by the local load distribution stresses.

The question might now be raised, "Is Professor Rathbun's solution incorrect, and, if so, what about the tests described in Part IV?." The answer, it seems to the writer, is that the proposed solution is correct as far as it goes,

and that Professor Rathbun's tests prove only what they were intended to prove and no more. If the crown section is taken, for example, it is shown that three forces and three moments act on it, that is, T_x, T_y, T_z, M_x, M_y, and M_z. These stresses are resisted by the unit stresses developed in the material of the arch. No matter what distribution of these unit stresses is adopted, their summation over the section, or their resultants parallel to the three co-ordinate axes, and their moments about these axes, must be equal and opposite to T_x, T_y, T_z, M_x, M_y, and M_z, respectively. The tests described in Part IV give only the values of the resultants of the reactions and do not show of what unit stresses they are the resultants.

If a simple beam with a load at the middle were similarly tested, it would show that one-half the load is carried to each support. The vertical shear acting on any section would ther be equal to either reaction, but the distribution of the shear is another matter. The test would produce the same result in the case of a rectangular beam in which the maximum shear at the neutral axis is 3/2 times the average, or in the case of a deep plate girder, where the shear at any point in the web is practically equal to the average shear. Neither would the test of this simple beam throw any light on the problem which is solved in practice by saying that the wheel load on a railroad bridge is distributed over a certain number of ties; yet, the test would be correct and would prove what it was intended to prove—that the resultant reactions have a certain relationship to the applied load and that they satisfy the laws of equilibrium. The present case, it seems to the writer, is analogous. The tests described by Professor Rathbun in the present paper, those made on a plaster model and reported in a previous paper entitled, "Crown Stresses in a Skew Arch,"[11] and those made by him and Professor Beggs on a rubber model,[12] are all of the same type, that is, tests for resultant reactions. What is required now are tests to determine internal unit stresses and their distribution, and this applies equally to symmetrical and to unsymmetrical loads.

Part II.—Method of Computing the Unit Stresses.—The writer does not agree with Professor Rathbun's method of computing the unit stresses. It is not necessary to go into all the reasons for this disagreement in any great detail since the writer has proposed a method of his own,[13] and the interested reader can make the comparison for himself. Some of these differences, however, will be discussed.

The first step in analyzing a structure of any kind is to find the total or resultant forces and moments acting on the various critical sections. In the case of the skew arch, it has been found convenient to do so on sections cut by radial planes passed through axes parallel to the abutments (Z-axes). Having done this, Professor Rathbun then proceeds to transform these forces and moments into other forces and moments in the direction of and about axes, which are parallel and perpendicular to the faces of the bridge. In Fig. 27, the forces, T_x and T_z, are shown transformed into t and s, and the

[11] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.
[12] *Loc. cit.*, Vol. 88 (1925), p. 1208.
[13] "Design of a Reinforced Concrete Skew Arch," *Proceedings*, Am. Soc. C. E., January, 1931, Papers and Discussions, Part II, p. 40, etc.

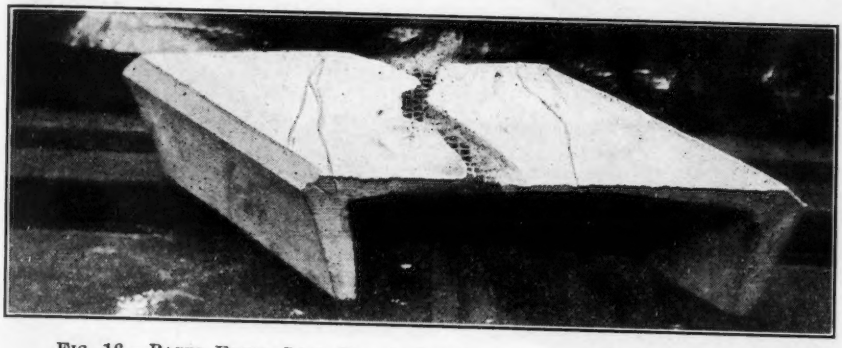

FIG. 16.—BASES FIXED, LOAD PLACED SYMMETRICAL, AND AT CENTER OF SPAN.

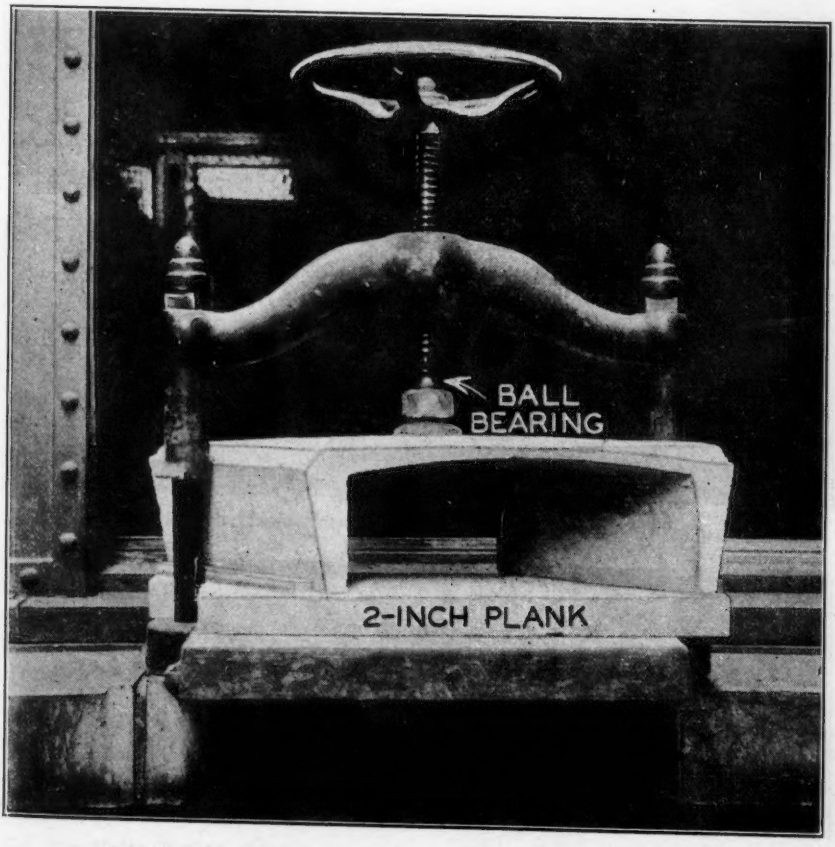

FIG. 17.—TESTING APPARATUS, SHOWING LOCATION OF REINFORCEMENT.

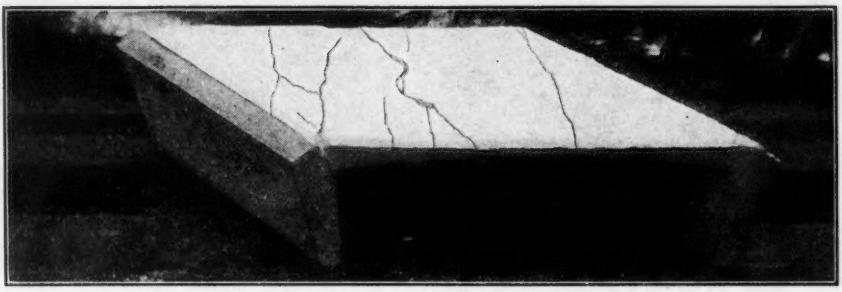

FIG. 18.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT CENTER OF SPAN.

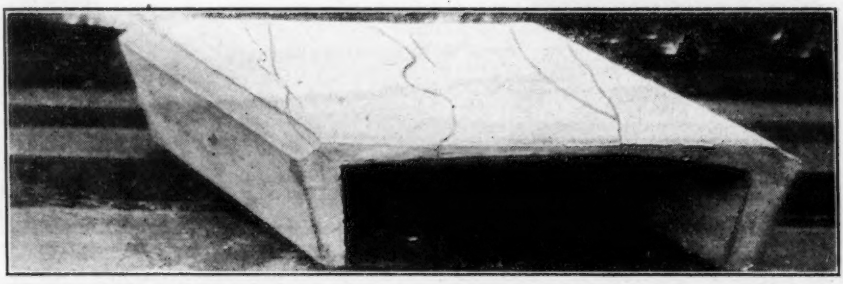

FIG. 19.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT CENTER OF SPAN.

FIG. 20.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

FIG. 21.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

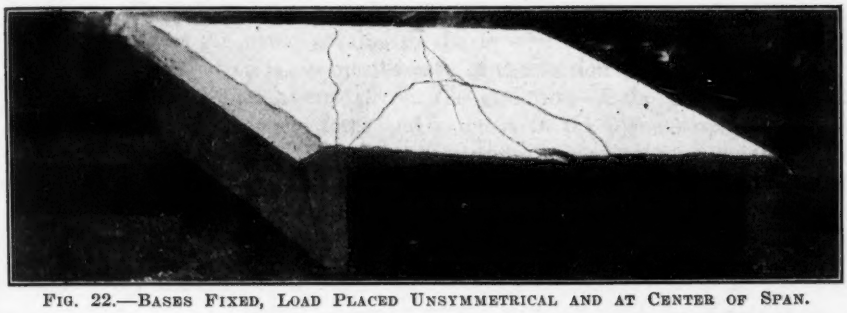

FIG. 22.—BASES FIXED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

FIG. 23.—BASES FIXED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

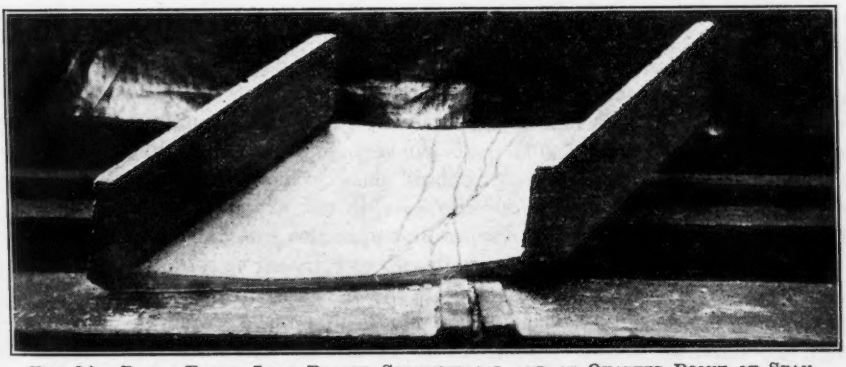

FIG. 24.—BASES FIXED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

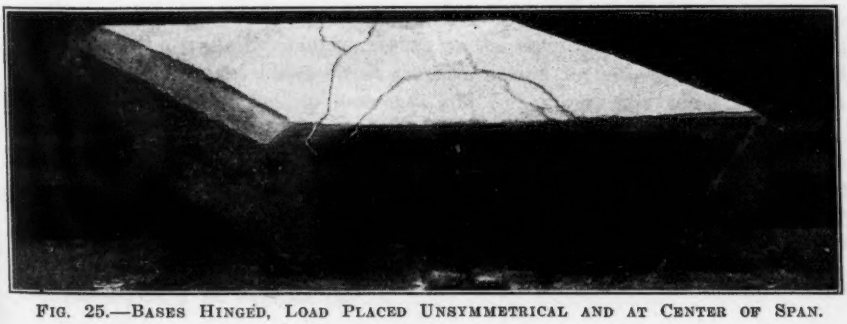

FIG. 25.—BASES HINGED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

moments, M_x and M_z, into m and f. The author then assumes that the force and moment in the plane parallel to the face of bridge, t and m, act as a thrust and moment on the projected area of the section on a radial plane perpendicular to the face of bridge (in the direction of the plane, AC). He similarly assumes that the force and moment in the plane perpendicular to the face of bridge, s and f, act as a shear and a torsion moment on this same projected area. No proof is given for this assumption and as will be shown later, it leads to incorrect results. If the zig-zag line AB (composed of segments parallel and perpendicular to the face of bridge), and the dimensions,

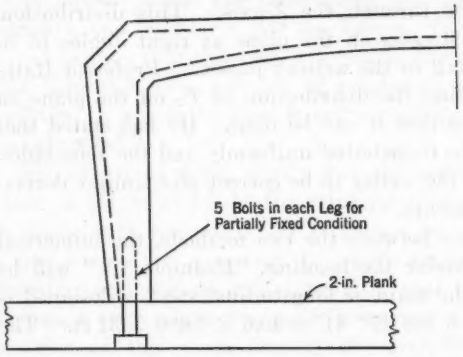

FIG. 26.

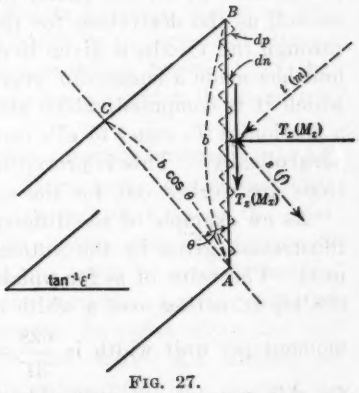

FIG. 27.

dp, dn, are considered, it might be argued that t and m produce tension and compression on the segments, dn, and shear on the segments, dp. Similarly, it might be argued, that s and f produce tension and compression on the segments, dp, and shear on the segments, dn. This is what actually happens, and the exact proportion of each kind of stress in each direction can be found. It does not make any difference in which direction the segments are taken, provided they are perpendicular to each other, thus giving a means of obtaining the stresses in any two directions at right angles to each other.

The operation which is given mathematically by the author's Equations (7), is merely a transformation of axes, but what is required is something entirely different. Textbooks on mechanics[14] derive formulas for finding the unit stresses on any plane when the unit stresses on any other two planes at right angles to each other are known. These formulas, when referenced to the principal planes, result in the equation of what is known as the ellipse of stress. These stress transformation formulas, or the ellipse of stress, apply equally well to moments and forces per unit width as to unit stresses.

A complete set of these equations in terms of the skew arch notation adopted by the writer is given in the paper previously mentioned[15] and need not be repeated. Their use requires that the total or resultant forces and

[14] "Strength of Materials," by Timoshenko, Pt. I, Chapter II; "Applied Mechanics," by Fuller and Johnson, Chapter II; "Theory of Elasticity," by Prescott, Chapter I, etc.

[15] "Design of a Reinforced Concrete Skew Arch," *Proceedings*, January, 1931, Papers and Discussions, p. 88, Equation (53).

moments on a section shall first be reduced to forces and moments per unit width, and that the stresses per unit width, if any, acting on a plane at right angles to the cut section shall be found. With these data, the stress per unit width on a plane making any angle with the cut section, and on the plane on which the maximum stresses occur, can be computed. This procedure gives a different result from that obtained by the method outlined by the author.

The writer's method based on the ellipse of stress requires that the total forces and moments first be distributed on the plane on which they are computed, that is, on the radial plane through the Z-axis. This distribution, as well as the derivation for the stresses on the plane at right angles to it, through the V-axis, is given in detail in the writer's paper.[16] Professor Rathbun has made a suggestion regarding the distribution of T_z on the plane on which it is computed, which shows that it can be done. He has stated that a portion of T_z equal to ϵT_x can be distributed uniformly and the remainder, parabolically.[17] This is proven by the writer to be correct and similar derivations are worked out for the moments.

As an example of the differences between the two methods, the numerical illustration given by the author under the heading, "Example 1,"[18] will be used. The value of m for which the main or longitudinal steel is designed is 628 kip-ft. acting over a width of $b \cos 47° 41' = 45.6 \times 0.680 = 31$ ft. The moment per unit width is $\frac{628}{31} = 20.2$ kip-ft. The writer obtains two values for this moment per unit width: One at the face of bridge and another for the central portion. The first value equals $\frac{M_z}{b} \sec^2 47° 11' = \frac{534}{45.6} \times 2.16 = 25.3$ kip-ft. The moment at the face of bridge is chosen because the proof of its correctness[19] is simple and is not subject to the uncertainty surrounding the correct formula to use for the torsion moment. The torsion moment does not enter into this part of the problem in the solution proposed by the writer.

Incidentally, Professor Rathbun uses Merriman's torsion formula whereas the writer uses St. Venant's formula. This is a matter of minor importance as neither is correct, as explained in the writer's paper previously mentioned. As far as the writer has been able to discover from the available literature on this subject, there is no derivation which applies exactly. St. Venant's formula is the most accurate, and for large ratios of $\frac{b}{t}$, that of Merriman is the least accurate.[20] Unless a correct formula, when developed, reveals something quite unexpected, its results need only be substituted for those used herein, and the discussion remains the same.

[16] *Proceedings*, Am. Soc. C. E., January, 1931, Papers and Discussions, Equations (35), (36), (43), (44), (48), (49), (51), and (52), p. 88.
[17] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 678.
[18] *Proceedings*, Am. Soc. C. E., April, 1931, Papers and Discussions, p. 526.
[19] *Loc. cit.*, January, 1931, Papers and Discussions, p. 45, Equation (34), and p. 89, Equation 53(*d*).
[20] *Bulletin No. 3*, Faculty of Applied Science and Eng., School of Eng. Research, Univ. of Toronto, Toronto, Ont., Canada.

The author has attempted to transform the computed moments and thrusts, T_x, T_z, M_x, and M_z, into the stresses, t, s, f, and m, respectively, because they are in the directions in which the steel is to be placed. This transformation is convenient, but the writer questions its correctness. Sections should be designed for the maximum moments and thrusts. The writer has referred previously to the stress transformation formulas which are a form of the equation for the ellipse of stress. These show that the maximum moments occur on one of the principal planes. The directions of the principal planes are determined by the relative values of the various moments, and, similarly, for the forces. It therefore happens, except at the face of bridge, that the steel will not be perpendicular to the principal planes. This fact requires a new set of reinforced concrete design formulas for the condition in which the steel is not normal to the plane on which the moments are computed.[21]

The ellipse of stress shows that the maximum shearing stresses occur on a pair of planes at 45° to the principal planes. Here, again, the planes form an angle with the steel. It is shown by the writer[21] that where the steel is not normal to a section it resists shearing stresses and direct stresses (bending and thrust) simultaneously and the two types of stresses can be separated only where the steel is normal to the section. Leaving out the question of the correctness of the transformation formulas given by the author, the writer still does not agree with the design for torsion in "Example 1."[22] Besides the points already enumerated, no account is taken of the relative deformation of the concrete and the steel. Incidentally, in designing for the maximum or principal flexural stresses, the fact that the steel is not normal to the section and that, therefore, the concrete and the reinforcement do not deform in the same direction must be taken into account. The same applies to the shearing stresses.

The writer has found that if the longitudinal steel in a skew "Rigid Frame" is placed parallel to the faces of bridge, no transverse steel is required for skew arch action. The photographs (Figs. 16 to 25, inclusive) seem to prove this in a general way, since the failure cracks are perpendicular to the face of bridge. At the center, some of the cracks make a small angle with the perpendicular to the face of bridge, and this also is to be expected. In a design of a similar bridge (skew angle, 50°),[23] this angle is found to be 11° 50′ for loads symmetrical with respect to the faces of bridge. In a slab, it can be shown that for symmetrical loads, as understood herein, the principal stresses will be parallel to the face of bridge. Apparently, a rigid frame differs very little from a slab as far as a certain type of skew arch action is concerned. For other reasons, transverse steel should, of course, be used even in a right arch, but that is another matter entirely. The transverse steel can be placed parallel to the abutments (and, if found necessary, it

[21] *Proceedings*, Am. Soc. C. E., January, 1931, Papers and Discussions, p. 52, and p. 90 to p. 112, inclusive.

[22] *Loc. cit.*, April, 1931, Papers and Discussions, p. 527.

[23] *Loc. cit.*, January, 1931, Papers and Discussions, p. 66.

can be designed that way) for simplicity in detailing. By the writer's method, the steel can be placed and designed in any direction desired, and the stress computed at any point and in any direction.

Conclusion.—Professor Rathbun's mathematics and the experimental work done by him and by Professor Beggs, demonstrate the relation between the load and the resultant reactions.

The mathematical demonstration of the relation between the reactions and the internal stresses is covered in Part II of the writer's paper.[24] The proportioning of the sections and design of steel, as suggested in this paper, are sufficiently accurate for design purposes, although certain factors which have not been experimentally demonstrated enter into the derivations. This experimental verification should be made so that no doubt will remain as to the exactness of the theory developed, even though a design based upon it, may be adequate for all practical purposes.

G. D. HOUTMAN,[25] ASSOC. M. AM. SOC. C. E. (by letter).[25a]—The writer has observed one bridge of several arches built on a pronounced skew, in apparent disregard of skew-arch theory, which shows no extraordinary crack or blemish. On the other hand, he has seen a lesser arch, built along lighter lines, showing a peculiar crack near the acute corner, as noted in Fig. 28. The direction of this crack is almost certain proof that the arch, near that crack, is not in compression, which confirms some of the current beliefs regarding skew-arch action.

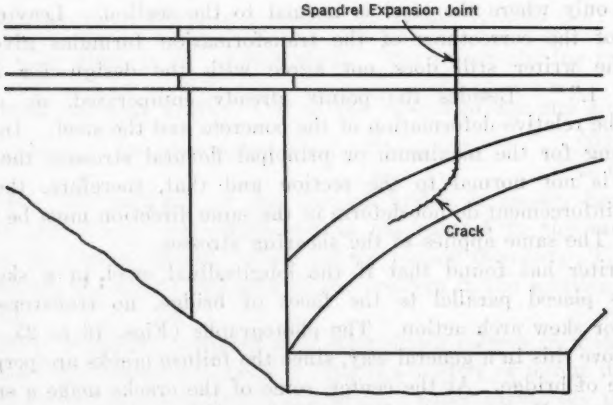

FIG. 28

[24] *Proceedings*, Am. Soc. C. E., January, 1931, Papers and Discussions, p. 40.
[25] Designing Engr., County Engr.'s Office, Delaware County, Media, Pa.
[25a] Received by the Secretary, April 30, 1931.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RUN-OFF—RATIONAL RUN-OFF FORMULAS

Discussion

BY LEROY K. SHERMAN, FRANCIS BATES, AND JOHN W. RAYMOND, JR.

LEROY K. SHERMAN,[12] M. AM. SOC. C. E. (by letter).[12a]—In Part I, the authors have presented an excellent analysis of the variable factors affecting the rates of run-off. Engineers at the present time are generally agreed that the empirical run-off formulas, based simply on a function of the drainage area, are of very limited usefulness. In Part II, all the numerous factors involved in the rational method for determining run-off have been grouped into a rational formula. In the writer's opinion this is not desirable. The rational method as a procedure is preferable to the rational formula.

Although it is somewhat lengthy, the rational method is a simple process of computation. The various steps follow in logical sequence and can be readily visualized. The relative effects of the various factors and approximations are apparent. In a formula involving seven variables these relative effects are concealed. In the design of a system of storm drains, the time spent in computation by the rational method is time well spent, and the substitution of a formula merely as a time saver is not warranted.

Certain steps in the rational method may be diagrammed or tabulated—just as the authors have done—to serve as a useful guide. The writer has prepared and used several such diagrams applicable to the rational method.[13] Other engineers have presented similar diagrams.

In Example 7, the authors have applied their formula to a drainage area of 500 sq. miles. The main stream has a length of 42 miles. If this stream has a channel of regular and uniformly increasing cross-sections, without enlargements or constrictions, then the rational method as applied to storm drains may be used.

NOTE.—The paper by R. L. Gregory and C. E. Arnold, Associate Members, Am. Soc. C. E., was published in April, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[12] Pres., Randolph-Perkins Co., Chicago, Ill.

[12a] Received by the Secretary, May 13, 1931.

[13] See, *Municipal and County Engineering*, June, 1924.

In general, natural streams draining large areas are irregular, with numerous enlargements and constrictions. On such streams the aforesaid rational method will not apply. This is because another variable (in addition to the seven mentioned by the authors) is involved; namely, the factor of pondage. In a stream with a flat slope, say, 1.5 ft. per 1 000, this pondage factor may reduce the peak rate of run-off 50 per cent.

For large drainage areas with unimproved natural channels the writer prefers the use of a rational method for determining run-off, based on the relation of an observed hydrograph to a known rainfall on the drainage area in question. This relation has been expressed as a coefficient by the Committee on Floods of the Boston Society of Civil Engineers[14] in its formula, $Q = C_F\sqrt{A}\,R$.

FRANCIS BATES,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—The authors present an original method for estimating storm-water run-off—based on fundamental conceptions and rational conclusions—which amplifies and refines the methods now in general use and should prove a valuable contribution to the literature heretofore published. The formulas are of sufficient flexibility to be made applicable to most of the rainfall curves, and contain factors neglected in the empirical formulas, but which, most certainly, are pertinent. They should prove very useful for determining the probable maximum flood at selected points, and for preparing preliminary cost estimates of drainage systems. The greater value of these refinements, however, should be in connection with the reduction of field observations made for the purpose of arriving at the correct values of the ratio of run-off to current rainfall, and, perhaps, the co-relating of the so-called values of C in the various empirical formulas.

Before the formulas and the numerical values of the new factors, P, and F, can be used with confidence, they must be subjected to close scrutiny and to comparison, as to results, with the experiences of others. It is not, however, the present purpose of the writer to examine the derivation of these formulas, nor the calculations, in detail, but rather to attempt, by their application, to show wherein may lie their greater value; and by questioning certain statements and conclusions made in the paper, to encourage the authors, and perhaps others, to clarify and enlarge upon certain phases of the presentation.

On first study of this paper, the reaction is that the subject-matter is very involved in assumptions, and the formulas appear to be complicated to a degree that would make their use difficult by the average individual, in general practice. However, on more thorough examination, it is clear that they are not so complicated once the type of curve is selected, and that they may be used in the same manner as the ordinary empirical formula when it is desired to take advantage of the extra factors submitted by the authors.

[14] *Journal*, Boston Society of Civil Engineers, September, 1930.

[15] Cons. Engr., Los Angeles, Calif.

[15a] Received by the Secretary, June 17, 1931.

These formulas could be made of especially practicable value if they were universally used in the reduction of field observations made to determine the proper values for C. A careful choosing of the type of water-shed, of correct values of P, F, and (hydraulic) S, and their application in the formulas, are not of such moment to the practitioner as is the choice of the elusive coefficient, C. These values are, as yet, so much a matter of judgment of the individual, based on his experience with present methods, that a change might easily leave one hopelessly undecided as to the proper choice. The introduction of the factors, P and F, results in an entire elimination of the former partly automatic correction of C values, and, unless the value now is chosen with extreme care, the advantages of the introduction are nullified. For instance, under the heading, "Method of Arriving at Values of Factor, P: Remarks and Deductions," it is stated that:

"For all practical purposes, therefore, the values of P (in Tables 3 and 4), shown opposite $\frac{A}{A'} = 262\,144$, and in the column headed by $P' = 0.5$, may be used when the formulas are applied without separate consideration of the inlet area, or nucleus."

These values of P have a range of from 0.42 to 0.62, and their significance in Equation (27), being $P^{0.5714}$, the actual values would lie between 0.61 and 0.76. If a constant value were assumed, such that $P^{0.5714}$ lay midway between these extremes, say, 0.685, the maximum error (either plus or minus), would be not more than about 11 per cent. One questions whether such refinement is warranted in everyday practice, at least until such time as the C values can be chosen more accurately. When using this same Equation (27), if one happened to pick 0.25 as a proper C value, whereas the correct one might be less or greater, say, 0.20 or 0.30, the erroneous assumption would result in a discrepancy of about 29 per cent over design, or about 19 per cent under design, as the case might be.

The formulas are developed logically and in a manner easily understood, except with respect to the factor, P. This appears first (see Equation (19)), as a coefficient of N, an abstract number found by inspection to be practically constant for any definite shape of channel with a given slope and Kutter's n. The next mention occurs directly under "Method of Arriving at Values of Factor P," wherein P is defined as the ratio of the average velocity of flow (through the water-shed) to the velocity at the outlet. In developing the formula, N is eventually replaced, in part, by the factor, F, representing the varying shapes and conditions of channel; and, in part, by the factor, P, as being representative of the ratio of internal velocities. It is not made very clear, however, why this factor should have this numerical value, nor why this final velocity has any bearing whatsoever.

However, if the results obtained by the application of the new formulas are found to be consistent with experience, they may well be accepted, at least in principle. The writer, for one, is not ready to concede unsoundness in all the empirical formulas, but will admit, only, that engineers sometimes use them unintelligently. These basic formulas may, then, be used as "yardsticks"

and by simple comparison of the old with the new, one may consider whether or not the principle of the latter is correct, not necessarily because of the theory expounded, but because of its consistency with the former, in results.

In the following, it should be kept in mind that the C of this paper is presumed to be the ratio of actual run-off to current rainfall, whereas the C of the empirical formulas (hereinafter called C'), is merely a correcting factor, inserted for the purpose of adjusting the equation to prevailing conditions. The more important of these empirical formulas in general use,[16] re-arranged so as to be in similar general form, may be stated, as follows:

Hawksley:

$$Q = C' R_1^{0.750} A^{0.750} S^{0.250} \quad (61)$$

Adams:

$$Q = C' R_1^{0.833} A^{0.833} S^{0.083} \quad (62)$$

Burkli-Ziegler:

$$Q = C' R_1 A^{0.750} S^{0.250} \quad (63)$$

McMath:

$$Q = C' R_1 A^{0.800} S^{0.200} \quad (64)$$

Parmley:

$$Q = C' R_1 A^{0.833} S^{0.250} \quad (65)$$

Hering:

$$Q = C' R_1 A^{0.833} S^{0.270} \quad (66)$$

Charles E. Gregory:

$$Q = C' R_1 A^{0.900} S^{0.186} \quad (67)$$

and, for comparison, Equation (29) may be written:

$$Q = 15 (FB)^{1.1429} C^{1.1429} R_1^{1.1429} A^{1.1429} S^{0.2143} \quad (68)$$

Equation (68) is a special reduction of Equation (24), with $i = \frac{K}{\sqrt{t}}$

Equations (61) to (68) show readily that identical results could not be obtained, from the use of any two or more of these formulas, except by a proper adjustment of the C values. There is no doubt but that each of these formulas gave results entirely satisfactory to the originator; but they proved satisfactory only because he was able to apply, intelligently, the C' values peculiar to the water-sheds upon which his observations were made and to the type of storms encountered. If these particulars were generally known, in detail, the formulas could probably be reconciled, and the mass of data, heretofore obtained, could be co-related.

An approximation can be made, which will give some idea of the relative C values, as used in the formulas of this paper, and those of one of the empirical formulas. The McMath formula (Equation (64)) is probably more widely used than any one of the others. Generally, it has been found to give fairly satisfactory results for the smaller areas, say, up to 1 000 acres, under conditions of natural flow; that is, without an improved drainage system. Assume, then, that the McMath formula and the authors' Equa-

[16] "American Sewerage Practice," by Metcalf and Eddy, First Edition, Vol. 1, p. 234.

tion (29), having $i = \frac{K}{\sqrt{t}}$, each give correct results, and equate them, as follows: Let $C' = C$, of the McMath formula, and $C = C$, of Equation (29). Then:

$$C' R_1 A^{0.800} S^{0.200} = 15 C^{1.1429} R_1^{1.1429} A^{1.1429} F^{1.1429} B^{1.1429} S^{0.2143} \quad \ldots\ldots (69)$$

and,

$$C' = 15 C^{1.1429} R_1^{0.1429} A^{0.3429} F^{1.1429} B^{1.1429} S^{0.0143} \quad \ldots\ldots (70)$$

Assume that $R_1 = 2.00$, the value originally used in the McMath formula, and that the value of S is between 10 and 100, then $R_1^{0.1429} = 1.104$; $S^{0.0143}$ lies between 1.034 and 1.068; and, taking the approximate mean of these values, $R_1^{0.1429} S^{0.0143}$ is called 1.160.

For natural conditions of flow, a value of $F = 4.20$ is chosen (Table 11) as being fairly representative, and $F^{1.1429} = (4.20)^{1.1429} = 5.156$. Substituting these representative values, Equation (70) now may be written:

$$C' = (15)\ (1.160)\ (5.156)\ C^{1.1429} A^{0.3429} B^{1.1429}$$

or,

$$C' = (89.71)\ C^{1.1429} A^{0.3429} B^{1.1429} \quad \ldots\ldots (71)$$

Again, with,

$A = 10$ acres, $A^{0.3429} = 2.202$;

$A = 100$ acres, $A^{0.3429} = 4.851$;

and,

$A = 1\,000$ acres, $A^{0.3429} = 10.683$;

and, for average conditions, assume that for 10 acres, $L = 1\,325$ ft.; for 100 acres, 4 220 ft.; and for 1 000 acres, 13 250 ft. In Table 3 under "$P' = 0.5$, Medium Concentration, $\frac{L}{W} = 3$", P is found equal to 0.48. This table is used rather than Table 4, because of comparison being made with the uniform assumption. Hence, values of $B^{1.1429}$ are: 0.01081, 0.00557, and 0.00290, for 10, 100, and 1 000 acres, respectively.

Then, substituting further, Equation (71) may be written for the three assumptions of acreage, as follows:

For 10 acres:

$$C' = (1.9422\ C)^{1.1429} \quad \ldots\ldots (72)$$

For 100 acres:

$$C' = (2.1700\ C)^{1.1429} \quad \ldots\ldots (73)$$

For 1 000 acres:

$$C' = (2.4458\ C)^{1.1429} \quad \ldots\ldots (74)$$

These formulas may be reduced and tabulated, for convenience of comparison, for the range of values generally used, such as is given in Table 12.

It must be remembered, however, that the comparisons in Table 12 are made on the assumption of drainage running in natural channels, or similar channels, in each instance. In case the flow is to be in a system of conduits, to obtain the relative values of C' of the McMath formula, for natural

TABLE 12.—COMPARATIVE VALUES OF C' OF MCMATH FORMULA AND C OF EQUATION (29), UNDER AVERAGE CONDITIONS OF NATURAL FLOW

C' of McMath	C of Equation (29)			C of Equation (29)	C' of McMath		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.069	0.062	0.055	0.10	0.154	0.174	0.200
0.15	0.098	0.088	0.078	0.15	0.244	0.277	0.318
0.20	0.126	0.113	0.100	0.20	0.339	0.385	0.442
0.25	0.153	0.137	0.122	0.25	0.438	0.497	0.570
0.30	0.180	0.161	0.143	0.30	0.539	0.612	0.702
0.35	0.206	0.184	0.163	0.35	0.643	0.730	0.837
0.40	0.231	0.207	0.183	0.40	0.749	0.851	0.975
0.45	0.256	0.229	0.203	0.45	0.857	0.973	1.116
0.50	0.281	0.251	0.223	0.50	0.967	1.098	1.259
0.55	0.305	0.273	0.242	0.55	1.078	1.224	1.403
0.60	0.329	0.295	0.262	0.60	1.191	1.352	1.550
0.65	0.353	0.316	0.281	.....	.....	.....	.....
0.70	0.377	0.337	0.299	.....	.....	.....	.....
0.75	0.400	0.358	0.318	.....	.....	.....	.....
0.80	0.424	0.379	0.336	.....	.....	.....	.....
0.85	0.447	0.400	0.355	.....	.....	.....	.....
0.90	0.470	0.420	0.373	.....	.....	.....	.....
0.95	0.492	0.441	0.391	.....	.....	.....	.....
1.00	0.515	0.461	0.409	.....	.....	.....	.....

channel flow, and C, of Equation (29), for improved channel flow, the appropriate F value (see Table 11) becomes a correcting factor. For instance, if the improved channel is to be circular pipe, with a value of Kutter's $n = 0.013$, the F value would be 6.63, and the relative values, as given in Table 12, would be corrected as follows:

(*a*).—With C' (of McMath) as a base: C (of Equation (29)) would be $\frac{4.20}{6.63} = 0.6335$ of that shown.

(*b*).—With C (of Equation (29)) as a base: C' (of McMath) would be $\left(\frac{6.63^{1.1429}}{4.20}\right) = \frac{8.688}{5.156} = 1.6850$ times the values shown.

For convenience of comparison, these values are also reduced, and tabulated in accompanying Table 13.

The values given in Tables 12 and 13, of course, are based on data presumed to apply to average conditions and for a particular type of rainfall curve. Those of Table 12 would be applicable to sub-areas with storm-water running loose to the catch-basins, while those of Table 13 have reference to

a drainage basin as a whole, with improved channels thoroughly covering the area. Similar tables could easily be prepared from any set of known conditions (or assumed conditions), in combination with any one of the empirical formulas. Using the McMath formula, with these tabulated values of C, the run-off, Q, was calculated for several of the illustrative problems given in the paper, and also for several drainage systems heretofore designed.

TABLE 13.—COMPARATIVE VALUES OF C' OF McMATH FORMULA, UNDER AVERAGE CONDITIONS OF NATURAL FLOW, AND C OF EQUATION (29), UNDER AVERAGE CONDITIONS OF FLOW IN CIRCULAR PIPES

C' of McMath	C of Equation (29)			C of Equation (29)	C' of McMath		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.044	0.039	0.035	0.10	0.259	0.293	0.337
0.15	0.062	0.056	0.049	0.15	0.411	0.467	0.536
0.20	0.080	0.072	0.063	0.20	0.571	0.649	0.745
0.25	0.097	0.087	0.077	0.25	0.738	0.837	0.960
0.30	0.114	0.102	0.091	0.30	0.908	1.031	1.183
0.35	0.131	0.117	0.103	0.35	1.083	1.230	1.410
0.40	0.146	0.131	0.116	0.40	1.262	1.434	1.643
0.45	0.162	0.145	0.129	0.45	1.444	1.640	1.880
0.50	0.178	0.159	0.141	0.50	1.629	1.850	2.121
0.55	0.193	0.173	0.153	0.55	1.816	2.062	2.364
0.60	0.208	0.187	0.166	0.60	2.007	2.278	2.612
0.65	0.224	0.200	0.178	.....	.....	.....	.....
0.70	0.239	0.213	0.189	.....	.....	.....	.....
0.75	0.253	0.227	0.201	.....	.....	.....	.....
0.80	0.269	0.240	0.213	.....	.....	.....	.....
0.85	0.283	0.253	0.225	.....	.....	.....	.....
0.90	0.298	0.266	0.236	.....	.....	.....	.....
0.95	0.312	0.279	0.248	.....	.....	.....	.....
1.00	0.326	0.292	0.259	.....	.....	.....	.....

This resulted, in each instance, in a value for Q approaching very closely that obtained either by the use of Equation (29), or by the detail method. Furthermore, for the particular conditions given, Tables 12 and 13 show C values fairly consistent with those usually assumed. Similar comparisons, made under extremely differing conditions, give equally satisfactory results. It may be presumed, therefore, that the formulas of this paper are of correct construction.

Throughout the paper, the presumption is that it would be incorrect to assume the individual storm as following the law of the rainfall curve; and yet, inconsistently, such storm is assumed to follow a definite law, that

of uniform intensity throughout any one time interval, with the measure of that uniformity as the average for that period. True, an individual storm might not, and usually does not, follow any known law, with respect to sub-intensities; but the improbability is just as true in one case as in the other. It must be remembered that rainfall curves are, at best, probability curves based on past experience, and that the tabulated i-values are average values, for certain time intervals, which are likely to be equalled or exceeded not oftener than the stated term of years.

If the designer is unwilling to concede the individual storm as following the law of the rainfall curve, he has no authority to assume his i-values from that curve. The two are inseparable, and to use them otherwise would be as inconsistent as designing a wooden structure with working stresses based on those of steel. Admittedly, a condition of uniformity would not occur as frequently as certain equal averages, in which case, if the former is assumed, the design would not be for the expected frequency, but for a much longer term.

Any one storm may have sub-intensities which are increasing in value, are uniform, or are decreasing, throughout concentration time, and yet may have a certain average value for that interval, hence, apparently tending to produce a maximum run-off greater than, equal to, or less than, respectively, that estimated under the uniform assumption. Under such variation, however, the concentration time would change, the tendency being for this time interval to decrease, in the order in which the conditions are stated, and the consequent adjustment of i would tend, automatically, toward a stabilization of the Q value, in conformity with that produced by the storm which follows the law of the curve. This condition is recognized in the assignment of the time of concentration of the nucleus, by assuming the maximum intensity as occurring at the beginning of the storm. Also, it is recognized in the statement, attributed to C. E. Ramser, M. Am. Soc. C. E., under "Use of Rainfall Curves," which is no doubt true and is probably universally accepted, namely:

"The maximum rate of run-off, therefore, would result from a rainfall of maximum uniform intensity, continuing for a time equal to or exceeding the time of concentration."

One can but interpret this statement to mean what it says: The maximum will be the greatest under the uniform assumption; and, this statement being true, the tendency, with respect to the effect of sub-intensity variation, must be as stated heretofore. Much of the confusion that exists, with reference to C, is caused by the use of i-values of average intensities, from these rainfall curves, as if they were a set of uniform intensities, and by the attempts to include with C an additional factor which must vary with the type of curve. As valuable as such a set of i-values would be, no feasible method has yet been found for compiling a curve which would give duration frequencies of uniform intensities; and engineers, therefore, must get along with what they have and make the proper corrections. All rainfall curves thus far developed give these i-values of average intensities rather than uni-

form, notwithstanding that the late Emil Kuichling, M. Am. Soc. C. E., as long ago as 1905, called attention to the necessity for maximum rainfall data based on uniform instead of average intensities.[17]

While the routine of the detail method requires the assumption of uniformity, this is done merely for convenience and as a practicable base from which to work. Otherwise, the conception would become so involved as to render its use impossible. Recognition of the inconsistency is given, however, through the medium of the assigned values for C. In no other circumstance could one account for the comparatively small values assigned to this coefficient, nor its relatively constant value, along with changing value of intensity. If $(1 - C)$ represents only the loss by absorption, evaporation, and retention, this loss would be rather constant in volume, per unit of time, and one could not justify this volume as being a constant percentage of a variable intensity; nor could one justify the apparent large loss to current run-off. On the other hand, if $(1 - C)$ represents not only the actual losses, as stated, but also a correcting factor, for the fictitious assumption of uniform intensity, the value of C would be relatively small, and would tend toward constancy with a variable i, in such ratio as the weight of the correction.

Undoubtedly, these values of C, as used, are not the proportions of the run-off to the actually controlling sub-intensities, but are the ratios that they bear to the average intensities, arbitrarily assumed to be uniform. The fundamental relation, $Q = C\,i\,A$, is a true expression when, and only when, the factor, i, is the one actually controlling. When the values given this factor are fictitious (and it is necessary that this be done because no better data are available), a qualifying coefficient of i could well be introduced, instead of making it a part of C, and the fundamental relation might be more properly written, $Q = C\,(ci)\,A$. In such an expression, C would have its true meaning, as generally understood and corresponding with the nomenclature of this paper; and the value of c would be 1.00 when the actual rainfall is of uniform intensity for concentration time, less than 1.00, otherwise, the actual value depending on the type of rainfall curve and other prevailing conditions.

Although this factor, c, would be a coefficient of i, it is, hereinafter, so that it will not be confusing, used in conjunction with C' (as in the empirical formulas), or $C'c = C$ (as in this discussion). With c having a maximum value of 1.00, for a rainfall of uniform intensity, probably the maximum values for curves of the type, $i = \frac{K}{t^e}$, on very small sub-areas, would follow, somewhat, the logarithmic mantissa of $(1.00 - e)$. With $e = 0.4$, 0.5, and 0.6, respectively, these maximum c-values would be log (6), log (5), and log (4), or 0.778, 0.699, and 0.602. The minimum value, for the very small sub-area, would be the ratio of the end sub-intensity to the average intensity, for the respective type of curves, or 0.60, 0.50, and 0.40.

In order to allow visualization of the foreging statements, the c-values, which equal $\frac{C}{C'}$, are given in Table 14, from the values shown in Table 12,

[17] *Transactions*, Am. Soc. C. E., Vol. LIV (1905), p. 192.

which values were determined from purely arbitrary assumptions of average conditions. Additional values are given, to clarify statements to be made presently, which were calculated simply by extending the scope of Table 12. In the McMath formula, however, when C' is given a value greater than unity, C' is no longer representative of approximately the degree of imperviousness, but is increased for other reasons. Therefore, in Table 14, for this higher range of C'-values, c would equal $\frac{C}{1.00}$ instead of $\frac{C}{C'}$.

A review of the method of making Table 12 and Table 14, would show, the other factors remaining constant, and for a rainfall curve of type, $i = \frac{K}{\sqrt{t}}$, that:

1.—With a change of C' (of McMath):

(*a*) C (of Equation (29)) varies as $\sqrt[1.1429]{C'}$.

(*b*) Within the range of $C' = 0$ to 1.00, c varies as $\frac{\sqrt[1.1429]{C'}}{C'}$.

(*c*) With C' larger than 1.00, c varies as $\frac{\sqrt[1.1429]{C'}}{1.00}$.

2.—With change of area size:

(*d*) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $(B^{1.1429} A^{0.3429})$.

3.—With change of intensity:

(*e*) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $R_1^{0.1429}$.

4.—With change of slope:

(*f*) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $S^{0.0143}$.

Assuming that all sub-areas, subsequent to the nucleus, have a concentration time either equal to, or less than, that of their respective contributing inlets (which must be, under the conceptions of the detail method), the controlling i-factor will be that occurring subsequent to the time of fall of that "remote drop" which is destined to join the "remote drop" from the nucleus; that is, if the t of the inlet is 60 min. and that of a certain sub-area is 20 min., the measure of the run-off from that area, in the maximum concentration from the whole, will be the actual intensities occurring during the interval (60–20) to 60 min.; and could not be the intensities prevailing during the interval, zero to 60 min. If, subsequent to the (60–20) moment, just cited, the actually controlling intensities are less than those of the (0 to 60)-min. period (and the form of the generally accepted rainfall curve would warrant such an assumption), the maximum simultaneous run-off concentration, from this sub-area, could only be measured by the lesser rate of fall.

The concentration time at an inlet point being the timing of the "remote drop" from the nucleus, it can be seen that the corresponding factor, c, would be representative of the period from zero minutes to the end of the concentra-

TABLE 14.—VALUES OF FACTOR c $\left(\text{WITH } i = \frac{K}{\sqrt{t}}\right)$

McMath C'	$c = \frac{C}{C'}$			McMath C'	$c = \frac{C}{1.00}$		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.6900	0.6200	0.5500	1.05	0.5373	0.4809	0.4267
0.15	0.6533	0.5867	0.5200	1.10	0.5597	0.5010	0.4444
0.20	0.6300	0.5650	0.5000	1.15	0.5819	0.5208	0.4620
0.25	0.6120	0.5480	0.4880	1.20	0.6039	0.5405	0.4796
0.30	0.6000	0.5367	0.4767	1.25	0.6259	0.5602	0.4970
0.35	0.5886	0.5257	0.4657	1.30	0.6477	0.5797	0.5144
0.40	0.5775	0.5175	0.4575	1.35	0.6708	0.5992	0.5316
0.45	0.5689	0.5089	0.4511	1.40	0.6911	0.6186	0.5488
0.50	0.5620	0.5020	0.4460	1.418	0.6990	.....	.....
0.55	0.5545	0.4964	0.4400	.....	.....	.....	.....
0.60	0.5483	0.4917	0.4367	....	.....	.....	.....
0.65	0.5431	0.4862	0.4323	....	.....	.....	.....
0.70	0.5386	0.4814	0.4271	....	.....	.....	.....
0.75	0.5333	0.4773	0.4240	....	.....	.....	.....
0.80	0.5300	0.4738	0.4200	....	.....	.....	.....
0.85	0.5259	0.4706	0.4176	....	.....	.....	.....
0.90	0.5222	0.4667	0.4144	....	.....	.....	.....
0.95	0.5179	0.4642	0.4116	....	.....	.....	.....
1.00	0.5150	0.4610	0.4090	....	.....	.....	.....

tion time, and, therefore, would be at its maximum value. If a sub-area, contributing at a certain inlet, had a concentration time equal to that of the inlet, the value of c, applying to the sub-area, would be this same maximum; but if the concentration time of the area were extremely short, the factor, c, would have a minimum value. The effect of each sub-area, on maximum simultaneous run-off from the whole, therefore, would be at its greatest when its concentration time equalled that of its tributary inlet; and at its minimum, when its t-value is only a small fraction of that of the inlet.

The relation between these two concentration times, therefore, becomes an important factor in the measure of maximum concentration. The shorter the t of the sub-area, the more frequent will be the turn-over of representative rainfall, and the controlling i factor will be that of the average intensity for the last frequency period of flow in which the "remote drop" from the area joins the "remote drop" from the nucleus. If t represents the concentration time of the sub-area, and T that of the inlet, the ratio, $\frac{T}{t}$, might be called the "turn-over frequency" of the area, and, in general terms, it may be stated,

that the larger the turn-over frequency of any sub-area, the less effect that area will have in the measure of final simultaneous run-off, the C value, consequently, being smaller with such increase.

Conceding this conception, one could visualize, as one extreme, a small impervious sub-area, with $t = 1$ sec., and T at its tributary inlet quite large. In such a case, the controlling i factor would be the current rate of fall, which with a curve of type, $i = \frac{K}{\sqrt{t}}$, would be one-half the average applying to the T of the inlet, or $c = 0.50$, and $C'c = (1.00)\ (0.50) = 0.50 = C$. As the other extreme, if $\frac{T}{t} = 1.00$, the controlling i factor of the area would be the same as that of the inlet, or $c = 0.70$, and $C'c = (1.00)\ (0.70) = 0.70 = C$. The range of C values, on small impervious areas, therefore, lies between 0.50 and 0.70, corresponding to the extremes of the turn-over frequency being infinitely large and of its being unity, respectively. On the other hand, in the case of a similar small sub-area, partly pervious, these limiting values of C would be different. The area of practically instantaneous run-off could have very little, if any, loss by absorption, evaporation, or retention, thus holding the C' value approximately at 1.00 (notwithstanding the area may be mostly pervious), and $C'c$ would be 0.50, as before. If $\frac{T}{t} = 1.00$, the area would sustain such losses, C' would be less than 1.00, and $C'c$ less than 0.70 (as estimated for wholly impervious areas). If the correct C' value were 0.10, and the corresponding c value slightly less than 0.70, say, 0.69, then $C'c = C = (0.10)\ (0.69) = 0.069$.

The range of C values, on small pervious areas, therefore, lies between 0.069 (or less), and 0.50, corresponding to the extremes of turn-over frequency being unity and of its being infinitely large, respectively. As the C' value increases from zero to 1.00, the corresponding c factor decreases from 0.70 to 0.50, and the product, $C'c$ (or C), would reach its maximum when $C' = 1.00$. From the foregoing, it may be noted, also, that C' (of the McMath formula) is not a measure of imperviousness of an area, but rather a coefficient expressing the effect of the combination of the degree of imperviousness and the ratio, $\frac{T}{t}$. It has a value of 1.00 when $\frac{T}{t}$ is infinitely large; decreases to zero as $\frac{T}{t}$ reaches unity and the degree of imperviousness reaches zero; and increases to its maximum value when the area is wholly impervious and the ratio, $\frac{T}{t}$, is unity.

These statements are made, specifically, with respect to the sub-area effect on the area as a whole, with concentration times of the sub-areas, and of the inlet points, being shown as controlling factors. Exactly the same conditions would apply to the sub-areas, considered independently, the minor subdivisions of these areas being substituted, and inlet points being considered

as collecting points for flow from the sub-divisions. The controlling factors, in such a case, are not so easily ascertained, because of the infinite possible variation of such minor concentrations. Resort must be had to judgment based on experience, and on the foregoing conceptions, for the correct assignment of these C values to the sub-areas. If the experiments of McMath are correctly reflected in his formula, and the average conditions he encountered were approximately those herein assumed, the C values of Table 12 would be approximately correct.

However, no pretense is made that the data in Tables 12, 13, and 14 are numerically correct, because the writer does not know the exact conditions under which the McMath observations were made. They do show, nevertheless, a remarkable consistency of trend, and of limiting values, with the foregoing analyses, and demonstrate conclusively that, not only do the detail method and the empirical formula assume the individual storm as following the law of the curve, but that this is true also of the formulas presented by the authors.

Many deductions, comparisons, and analyses of differing features could be made from the foregoing. In general, however, it may be stated that, with a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, any area characteristic which increases the turn-over frequency of its sub-areas will decrease the value of C and *vice versa*. An increase in sub-area size (T larger), of intensity (t smaller), and of slope (t smaller), will increase the value of the ratio, $\frac{T}{t}$, in consequence of which C values will decrease. The increase in degree of imperviousness (which decreases t), would also cause a change in the same direction, except that the trend is reversed by the decreasing ratio of actual loss, by evaporation, absorption, and retention. The actual variation of C values, caused by change in the foregong relations, is entirely dependent upon the type of curve. A few simple comparisons, made with other values for e, show clearly the reason for the differences among the various empirical formulas.

When the sub-areas are considered as part of the whole, their respective C values will remain constant only when the turn-over frequency of the area, or $\frac{T}{t}$, is unity. Otherwise, C must depart from these independently determined values, and will be more, or less, according to circumstance. As each sub-area is encountered along the course of the main channel—an increase in time—the ratio, $\frac{T}{t}$, would increase, of course, although t would increase slightly because of decreasing intensity. The latter may be disregarded, however, for any change to be made in the C value, because of the changing ratio, $\frac{T}{t}$, would be based from the original values of both C and t.

Theoretically, then, with increase in time, C would be reduced in such measure as the $\frac{T}{t}$ value departed from unity. Practically, however, this trend might be nullified, or even be reversed, if increasing wetness of the area increases the real value of C'. Especially in the lower bracket of values, if the original assignment of the C' value is based solely upon the percentage of impervious surface, without regard to what the condition of the pervious parts would be when thoroughly wet, the theoretical trend would most certainly be reversed; and C would have to be increased with increase of time. The underlying reasoning back of the original assignment, with respect to perviousness, wetness, size, and relative position of the sub-area, would determine whether C should, later, be increased, remain constant, or be decreased. Probably each of the authorities, who do one or the other, are correct, basicly, founding their action on the characteristics of the watersheds to which the applications are made, on the type of rainfall curve appertaining to his locality, and on his original assignment of the C' value.

The initial sub-area, or nucleus, becomes of importance in the detail method, not, primarily, because of its physical characteristics, but because of the interval chosen as its concentration time. This value of t is added to the calculated time in the drains, in order to arrive at concentration times of subsequent inlets, which, in turn, determine the respective intensities. Thus, the assumed t of the nucleus becomes a factor in the whole design, of greater weight if total time is short and of less consequence if it is large. With a curve of the type, $i = \frac{K}{\sqrt{t}}$, if 10 min. is chosen as initial inlet time, whereas 15 min. would be correct, the corresponding intensity values would be in error about 22%; and at the end of the total time of 1 hour, this error would be reduced to 5%; but this error would be nullified, except as to the nucleus, if due consideration is given to the C value change on subsequent areas. The ratio, $\frac{T}{t}$, for the latter, would be different with the erroneous assumption, and C values on these subsequent areas would change accordingly, the tendency being to offset the seeming error. Nevertheless, initial inlet time should be chosen as accurately as possible.

In their discussion in Part I, the authors observe and comment on many seeming inconsistencies in the empirical formulas, and in the assumptions of the detail method. Obviously, it would be inadvisable to comment on each of these, separately, because of the space required. Many of their queries are reconciled in the foregoing analyses, and many of the apparent inconsistencies turn out to be entirely logical. Some of the more important of these may be recalled, briefly:

In their "Introduction" the authors state that, "one purpose of the paper, however, is to show why the use of these empirical formulas is not now to be recommended," whereas, actually, their own theory may be established by the use of them.

The conclusions drawn from the illustration of the simple water-shed (see Table 1) are based upon the impossible assumption of unity for the C values, and that these values would remain constant under the stipulated conditions. The several columns of tabulated values of Q (Table 2), would hold only when the average intensities, shown for each sub-area, are considered as being uniform, and as continuing at their respective rates for the entire concentration time of the total area.

Preceding the authors' conclusions to Part I, the statement that "to increase Factor C as the time increases (as shown in current publications and textbooks), without also allowing for the fluctuation in C for changing intensities, seems entirely unjustified," cannot, itself, be justified. It has been shown, earlier herein, by using Equation (29), that the weight of i, in so far as it affects the factor, C, is $R_1^{0.1429}$, and that C varies inversely as this value.

It has been previously shown that, with N a constant for any set of conditions, $C' = NC^x$, and C, therefore, varies as $(C')^{\frac{1}{x}}$; also, since $C = C'c$, c varies as $\frac{(C')^{\frac{1}{x}}}{C'}$.

With $\frac{T}{t} = 1.00$, the ratio of actual intensity to average intensity is 1.00; and, with $\frac{T}{t}$ infinitely large, the ratio of actual intensity to average intensity is $(1.00 - e)$. The mean of these two extremes is $(1.00 - \frac{e}{2})$, and the equation of the curve for values of C and c is, therefore, the log of this mean, and $\frac{1}{x} = \log (1.00 - \frac{e}{2})$, and $x = \frac{1}{\log\left(1.00 - \frac{e}{2}\right)}$.

With respect to the effect of intensity on C values, the authors show that velocity varies as $Q^{\frac{1}{4}}$; hence, t^x varies as $\frac{1}{R_1^{\frac{e}{4}}}$, and the ratio, $\frac{T^x}{t^x}$, varies as $\frac{TR_1^{\frac{e}{4}}}{1}$. The equation for the curve, for the effect on C' values, is, therefore,

$$\frac{ex}{4} = \frac{e}{4\left(\log (1.00 - \frac{e}{2})\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

With a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, the exponent for the curve of C values, or x, is, $\frac{1}{\log (1.00 - \frac{0.5}{2})} = \frac{1}{\log 0.75} = \frac{1}{0.8750613}$; and $x = 1.14277$. The effect of intensity, on C' values, is $\frac{ex}{4} = \frac{0.5x}{4} = 0.1429$.

In a similar manner, it can be shown that the effect of the slope, on C' values would be, $\left(\frac{e - e^2}{2}\right)\frac{x}{10} = 0.0143$.

These are the exponents obtained by equating the McMath formula with Equation (29) to obtain Equation (70).

The authors are most vague as to their reason for inserting the factor, P, in Equation (19), but a study of its significance in the final expression (Equation (24)) would indicate this factor to be merely a measure of the effect of the ratio, $\frac{t}{T}$, on maximum run-off. In other words, the factor, P, is a measure of the reduction of C values (as assigned to the sub-areas, independently) required by the average value of the ratio, $\frac{t}{T}$. The latter, of course, would be different for each type of water-shed (see Fig. 2) and for each type of rainfall curve; but, also, these P-values (see Table 4) would change, for any one type of water-shed, or rainfall curve, if the slope of the area were variable, or if it differed considerably from that of the main channel, and if the C-values were different on the various parts, Table 4 being calculated with the assumption of uniform slope and a constant C-value. The suggestion is made that one table might answer the purpose better, wherefrom, with average $\frac{t}{T}$ -values for guides, P-values could be selected appertaining to the more generally used types of rainfall curves.

Table 3 is mis-named, in that the value of P has no significance when the storm is of uniform intensity. Under such condition, the value of e is zero, and any value of P, with zero for its exponent, has no meaning.

The illustration of the tabling process (Table 8) is useful for demonstrating the consistency of the suggested formula, but it might be very misleading with respect to the assigned values of C. Although the slopes are fairly uniform, this factor is assumed to be the same for the extreme variation of areas of 1.5 and 108.6 acres, respectively. It might be possible, but not very probable, that these sub-areas, individually, have equal C-values, but each, as a part of the whole, could not possibly have such equality.

In conclusion, it may be said that the paper presents many valuable and original suggestions and that the authors are to be commended for their really fine effort. Two additional factors are introduced which are of vital importance, and their adoption can only lead to a more precise determination of real C-values. A word of caution must be uttered, however, against the general use of these formulas without a thorough understanding of the effect, and importance, of the C-value assignment. The introduction of these two new factors carries with it the necessity for a very much greater refinement in these selections, which, if not done with proper knowledge, may easily lead to results more erroneous than if they had not received consideration.

JOHN W. RAYMOND, JR.,[18] ESQ. (by letter).[18a]—This paper discusses the problem of designing culverts, bridge openings, storm drains, spillway structures, etc., in which it is required to estimate maximum flow. The estimation of discharging capacity for bridge openings, culverts, spillway structures, and flood channels, while involving basic data very similar to those required for determining flows to be provided for in storm drains, is nevertheless a distinct problem. Since it is usually considered good engineering to provide for the maximum flow to be expected in the first case, in storm drain and combined sewer design, it is rarely practicable, financially, to provide conduits throughout a drainage area, to care for the maximum flows. Thus, the problem of storm drain or combined sewer design introduces the estimation of frequency and of balancing of damages and inconvenience resulting from occasional flooding, against the extra cost of providing conduits for the larger expected flows.

While both problems deal with structures which are continually in use, the problem of designing storm drains and combined sewers is to provide a system within the funds available and in agreement with the degree of protection required, and the rate of flow to be provided for is that expected at occasional intervals, say, on the average of once in 5, 10, or 15 years. The problem of estimating discharge capacity for culverts, bridge openings, spillway structures, etc., requires the determination of the maximum flow to be expected in more than 100 years, or perhaps even 1 000 years. Thus, the conditions for the design of the latter structures require the determination of the maximum rainfall conditions, together with the maximum run-off conditions, and the problem of storm drain and combined sewer design requires the determination of the rainfall to be expected at a given frequency and average run-off conditions, since the condition of simultaneous maximum or excessive rainfall and maximum run-off is of rare occurrence.

Considerable labor is involved in the tabling process as practiced by a number of engineers. For a system of storm drains or combined sewers to be designed by means of the Gregory and Arnold formulas, probably the most convenient way to arrange the computations would be in tabular form for the different sections of the conduit, substantially as is done in the tabling process for the rational method; and it seems to the writer that the number of columns and the labor involved would not be far different in either case.

It is not axiomatic that the maximum flow resulting from a rain of given frequency occurs when the entire area is contributing; it is generally true, especially for areas which increase substantially as the time of flow. It may not be true where a water-shed consists of large individual areas, the main channels of which have different slopes from that of the main channel for the whole water-shed or where the development or proportion of impervious surface of the individual areas is widely different. Engineers using the tabling process in the rational method of design should investigate their design for durations shorter than the time required for the entire area to contribute, to find the maximum flow resulting from the selected frequency rainfall curve.

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[18] Asst. Engr., Metcalf & Eddy, Boston, Mass.

[18a] Received by the Secretary, July 24, 1931.

It is true that intense rains seldom occur at uniform rates for long durations. It is believed, however, that with a complete system of storm drains or combined sewers, consisting of closed conduits, the use of uniform intensities of rainfall gives substantially correct results, on account of the limited capacity of the conduits. This causes water precipitated during the short periods of excessive rainfall to be retarded, thus smoothing out the run-off. If the rainfall curve is constructed from records of all storms, it may be used with reliability in determining the frequency, provided the records cover a sufficient number of years.

Where the proportion of impervious surface is uniform, the run-off factor, C, for a storm beginning at a time when run-off conditions on the area are average, or when the area is not completely saturated, is known to increase with time, and when this is taken into account the different values of Ci vary less than the i values.

The detailed method as practiced by most engineers is not based on the assumption of the equivalent of a variable intensity during the period of concentration, but upon the assumption of a uniform intensity for each period, or in other words, the storm used for each part of the drain is a different storm than those used for the other parts.

The usual method of construction or derivation of rainfall curves includes all significant storms, and in a record the length of which is several times the period of frequency represented by the design curve, there will be several storms which will define the position of the frequency curve. It is incorrect, therefore, to assume that the run-off at a given point resulting from a storm of a certain frequency, is made up from rainfalls of diminishing intensities, different intensities being applied to different parts of the drainage area, since the ordinate, i, equals the total fall divided by the duration.

The authors have presented a valuable discussion of the problem of estimating storm flows. It is difficult, however, for an engineer experienced in the use of the so-called "detail rational method" to visualize the bearing on the problem of the different coefficients and constants which are suggested. If the formulas were felt to be applicable, a great deal of experience in their use would be required, to obtain reliable results. On the other hand, where the detailed rational method is used for storm drain design, an experienced engineer may select the basic data, that is, the frequency rainfall curve, degree of development, and coefficients of run-off; and he may then explain the method of computation to a less experienced assistant, who can complete the labor of design, and reliable conclusions can be reached. It does not appear to the writer that the labor involved in working out the results of the suggested formulas can be entrusted to an inexperienced assistant to the same degree.

The authors have clearly pointed out the inadequacy of empirical formulas, and their continued future use for estimating the required capacity for important drainage channels and waterways can scarcely be tolerated.

Probably the "inlet time" as used in the rational method of storm drain design gives engineers the most concern because of the paucity of reliable data, and the need for more observations cannot be over-emphasized.

The distribution of intense rainfall over the area of a water-shed is an important factor. Excellent data for rainfalls of short duration have been presented by Frank A. Marston[19] M. Am. Soc. C. E. Fig. 8, which is from

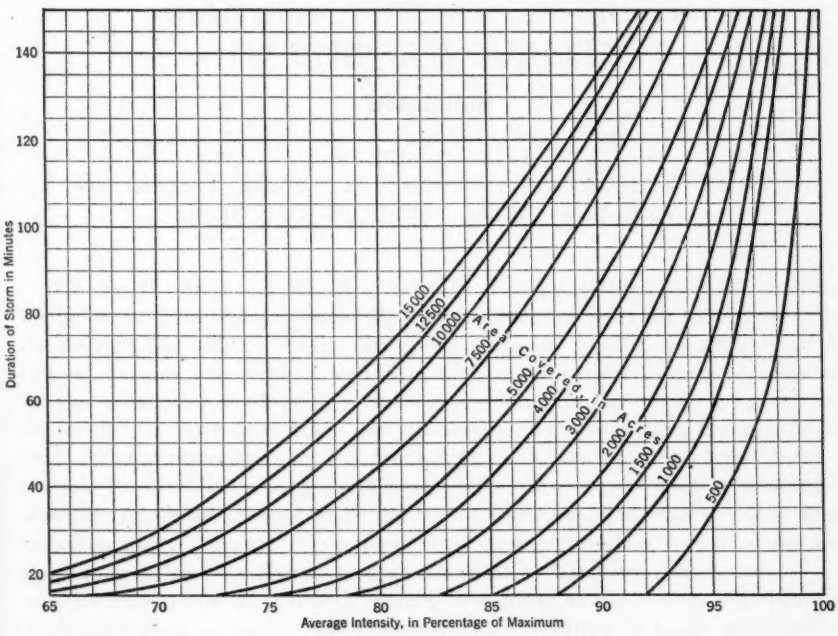
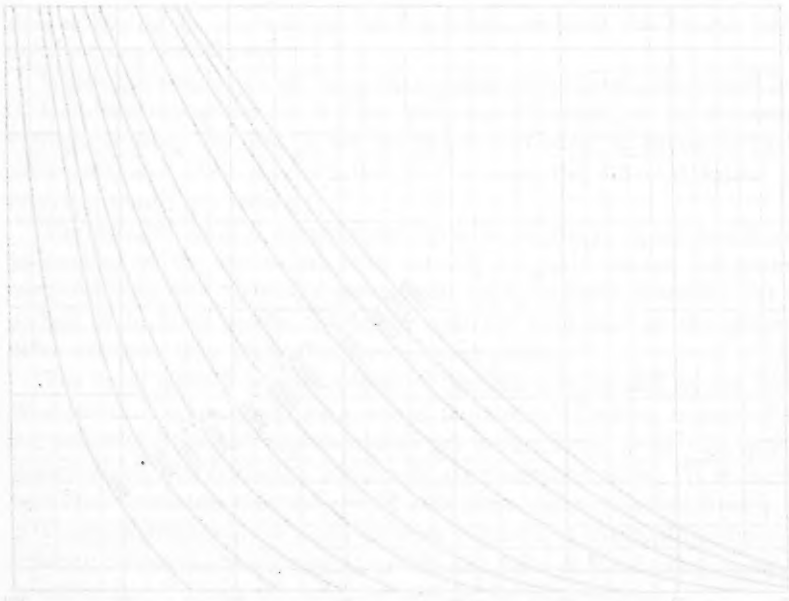


FIG. 8,—DISTRIBUTION OF RAINFALL INTENSITY WITH RESPECT TO AREA COVERED AND DURATION OF STORM FOR 15-YEAR FREQUENCY STORMS AT LOUISVILLE, KY.

the report of the Commissioners of Sewerage of Louisville, Ky., dated October 16, 1929 (and prepared from data in Mr. Marston's paper, and some subsequent data), shows the relation between the average and maximum intensities for different areas and durations.

19 *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 535.



AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MANUFACTURING CONCRETE OF UNIFORM QUALITY

Discussion

BY MESSRS. HENRY B. SEAMAN, THEODORE BELZNER, JOHN SANFORD PECK, AND J. W. KELLY

HENRY B. SEAMAN,[9] M. AM. SOC. C. E. (by letter).[9a]—With uniform materials and uniform methods of manufacture, such as those described by the author, one should obtain reasonably uniform results; a variation of 20 to 50%, as shown in Table 2, would seem to be as uniform as may be expected. In the construction of bridge masonry in large mass, however, more than uniformity is desired; solidity and durability of the structure are also necessary. Early strength may be misleading. It has not been considered a prelude to durability. Engineers have preferred a cement that is moderately slow-setting, with gradual and continuous increase of strength. Care must be taken not to "drown" the cement and produce laitance, but a moderately wet concrete has been found to give the most satisfactory results.

The importance of thorough mixing is as great as the quality and quantity of cement used, or of the water-cement ratio—whether it be 5½ or 6½ gal. of water per bag of cement—and usually much less than that will suffice. Then, too, a proper and complete distribution of aggregates in the forms is important.

Concrete of proper consistency for moulding will segregate in transportation, whether this is done by wheel-buggies, trucks, chutes, or blow-pipe. From wheel-buggies the fluent mortar will be deposited first and the coarse aggregate will follow, but from chutes or blow-pipe the coarse aggregate will be deposited first and the mortar will follow. This latter condition is the more desirable since, in spading, the mortar envelops the stone which rises, and the mortar flows down through the interstices. It is partly for this reason that chuted concrete, or blown concrete, has given such satisfaction in field work. Another, and an important, reason appears to be the fact that the chute, or blow-pipe, gives additional mixing to the mortar as it turns over while flowing.

NOTE.—The paper by William M. Hall, M. Am. Soc. C. E., was published in May, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[9] Cons. Engr., Brooklyn, N. Y.

[9a] Received by the Secretary, May 20, 1931.

Attention seems to be given, by some experimenters, to the preference of having the mixture so dry that it will not segregate during transportation, but such concrete will not fill the forms properly unless thoroughly rammed and compacted, and, even then, it has been found, upon removal of the forms, that voids have been left on the surface, and similar conditions may be expected in the interior of the mass.

For many years a 1:2½:5 mixture was generally used and was considered a theoretically perfect mix, but about 1900 the 1:2:4 mix was adopted and is now (1931) established as the general standard. Wastefully rich, it seemed at first, but experience has justified its adoption as giving assurance against imperfections that will creep into practical work of magnitude. The question that remains is how to obtain the best results with this mixture. Thorough mixing is the key to good concrete. If a single mixer will not produce concrete fast enough for an extensive job, tandem mixers, or even twin mixers delivering batches alternately, may be adopted. The extra cost will be insignificant on large work, and will secure a much more uniform and satisfactory product.

THEODORE BELZNER,[10] AFFILIATE, AM. SOC. C. E. (by letter).[10a]—The writer has read this timely paper and Appendices I and II with great interest. The excellent and painstaking work represented by the systematic and uniform method of inspection and control of concrete manufacture, authorized and carried out to the end, is highly commendable. The value of the results secured should serve as a strong incentive to other engineers to conduct similar investigations along these lines.

Furthermore, the absolute essentials necessary for securing uniform success in producing good concrete of uniform quality, resulting from the proper training, experience, and qualifications of those responsible for this class of construction in the field, cannot be too strongly emphasized. When structures of this class are so large and when the resources of construction in workmanship are taxed to a higher degree than usual, the results have far greater value than laboratory experiments could possibly give.

JOHN SANFORD PECK,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—This paper marks another milestone in the long struggle toward the general adoption of scientific and controlled methods of concrete manufacture in the field. Since the first promulgation of the water-ratio theory by Duff A. Abrams, M. Am. Soc. C. E., the battle to introduce his theories into practice has been a long and, at times, almost a losing one.

When Mr. Hall states, in his "Introduction," that more than 50% of the concrete made and placed to-day is of poor quality and will have a short life, he is erring strongly on the side of conservatism. The writer has yet to hear

[10] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Plant and Structures, City of New York, Brooklyn, N. Y.

[10a] Received by the Secretary, May 20, 1931.

[11] Instr. in Civ. Eng., College of the City of New York, New York, N. Y.

[11a] Received by the Secretary, May 25, 1931.

of any concrete being condemned or replaced because the test cylinders failed to meet the 28-day strength requirements; and the jobs on which sample cylinders are taken, constitute an almost infinitesimal proportion of the concrete work done in the United States. The use of "ready-mixed" concrete in large cities has gone a long step toward procuring concrete of reliable strength and durability, but even there, the room for improvement is vast.

As an example, the writer witnessed a mixer truck discharging ready-mixed concrete into the basement of a New York building. As the concrete left the mixer it seemed to be of good quality and consistency. At the top of the chute which led to the basement a workman stood with a hose, washing the concrete down. What became of the water-cement ratio in this case is easy to imagine. Such ignorance is a fair example of what still remains to be done in the line of education.

Of all the factors listed by Mr. Hall ("Irregularities in the Strength of Concrete"), which may affect the strength of concrete, the first, namely, the strengths of different brands of cement and the variation even in different parts of the same bin of a given brand, constitutes by far the most difficult variable to control. This wide variation has been long known in the laboratory. It is unfortunate that standard 2 by 4-in. compression cylinders were not included in the cement tests because it is the writer's opinion that these are a far better indication of the probable strength of concrete to be obtained from a given brand of cement than the tension briquettes. The compression tests as set forth in the American Society for Testing Materials Tentative Standards for 1930, Designation C 9-16T, should be adopted as a standard.

With the compressive strength of standard 1:3 cement mortars varying from 2 000 lb. per sq. in. (the American Society for Testing Materials minimum, which incidentally is too low) to 5 000 lb. per sq. in. at 28 days, the attempt to make concrete of a predetermined strength in the field is saddled with a severe handicap. The frequently expressed theory that all cements are equal seems absurd.

The account of the training and education of the field force of inspectors is worthy of more than passing comment. If the idea that special training and knowledge are necessary for the making of good concrete, could be impressed on the smaller contractors and job superintendents it might lead to the growth of a class of "concrete technologists" which in time would supplant the old-time concrete foreman or "mixer-man" who is supposedly endowed with a sixth sense whereby he knows all there is to know about concrete making.

The conditions under which the work described by Mr. Hall was done certainly were not of the easiest, and the fact that such satisfactory results were accomplished speaks volumes for the efficiency of his method of educating and training his inspection force. His experience in this regard should prove of great value in the planning of future work whether of the same magnitude or smaller, and it is to be hoped that his paper will establish a precedent for the construction industry as a whole to follow to the great benefit of the cause of good concrete.

J. W. KELLY,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a] As stated by Mr. Hall, the writer was to some degree associated with the men of the inspection force in the Louisville District during 1926, 1927, and 1928. One could not fail to be impressed with the size and variety of the project, and with the organization's policy of thorough training for the men in order to produce a uniform concrete.

An apparent lack of uniformity in the strength tests, as exhibited by the maximum and minimum range of values in Table 1, has been pointed out to the writer. However, it must be remembered that the table averages nine jobs, spread out for nearly 400 miles, with concrete made by different organizations using different types of equipment and a variety of cements, admixtures, and river-bar aggregates; also, that the percentage of test specimens which were considered defective and which were omitted from the report is much smaller than is usual on concrete work. Unquestionably, the record of each job would show a smaller spread of strength values. With specification requirements which would normally produce 1 700-lb. concrete, as stated by the author, it is evident that the average quality of the work was greatly improved through the efforts of the inspection force.

Mr. Hall's analysis of the causes of variation in concrete or in specimen strength is a clear indication of the chief features to watch in order to obtain uniform results in any concrete construction. Elsewhere in the paper he emphasizes the training of individuals, even the spaders and mixer men. This reinforces the writer's observation that the human element enters largely into concrete making, and that while the process is simple it requires intelligent and conscientious workmanship to bring a "good enough" job into the category of a first-class piece of work. First-class concrete can be made without undue effort, but too many organizations seem to be satisfied with the line of least resistance.

Improved construction practices have been strongly urged by the Portland Cement Association, particularly in regard to placing and curing. It is gratifying to note that the care taken by the construction forces on the Ohio River bear out these recommendations by their example. Improvements in the strength of cements during the course of the project are noted, as well as the practice of re-washing coated aggregates, which process would tend also to reduce variations in grading through the attendant mixing of sizes.

Inspection has too often been regarded as a perfunctory matter, tending to retard construction and to increase cost, whereas the right kind of inspection (whether for concrete or for any other construction) is one of the most worth-while investments that can be made. It is the writer's belief that qualified inspectors, preferably but not necessarily with formal technical training, are worth far more than their present scale of salaries indicates, and that their services are as real and valuable to the project as those of the

[12] Minneapolis, Minn.

[12a] Received by the Secretary, July 27, 1931.

designer or the construction force. More and better inspection would eliminate many of the costly mistakes, early maintenance, and construction of inferior quality that are now encountered.

After a long experience with this subject, Mr. Hall considers that sound concrete of uniform strength is best obtained by good workmanship. Better realization of this by all who deal with concrete in its many angles, is sure to result in better structures.

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DISCUSSIONS

STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK

Discussion

BY B. F. JAKOBSEN, M. AM. SOC. C. E.

B. F. JAKOBSEN,[58] M. Am. Soc. C. E. (by letter).[58a]—Before taking up the discussions, the writer wishes to restate his problem. This seems advisable, because several contributors, notably Professor Campus, have misunderstood the writer's reference to the classical theory. The writer had the ordinary engineering theory in mind, while Professor Campus had the classical theory of elasticity in mind. The writer did not properly distinguish between these two theories, and this led to the misunderstanding. An excellent and critical résumé of the classical and engineering theories has been given by Professor Campus in another paper.[59]

The Classical Theory of Elasticity.—This assumes a homogeneous, elastic, triangular section of indefinite length. It implies, as Professor Vogt has shown in his discussion,[60] that sections horizontal before loading are curved, (concave side upward), due to the loading. When this theory is applied to ordinary dams, which are necessarily of limited height, it implies as an assumption, that the foundation is thus curved by the loading, but this implied assumption is no part of the classical theory, which does not consider any foundation.

The writer has shown that the linear stress distribution satisfies the statical conditions, Equations (3) and (4), the equilibrium Equations (6) and (7), and the boundary conditions, Equations (8) and (9). That these stresses also satisfy the continuity equations was shown in Appendix II.

NOTE.—The paper by B. F. Jakobsen, M. Am. Soc. C. E., was published in September, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by the late William Cain, M. Am. Soc. C. E.; November, 1930, by Messrs. William Gore, L. J. Mensch, Johannes Skytte, and Lars R. Jorgensen; January, 1931, by Messrs. C. A. P. Turner, Donald P. Barnes, and Anthony Hoadley; February, 1931, by Messrs. Fred. W. Ely, and M. H. Gerry, Jr.; April, 1931, by Messrs. Fredrik Vogt and Eugene Kalman; and May, 1931, by Messrs. Fernand Campus, A. Floris, Robert E. Glover, and P. Wilhelm Werner.

[58] Cons. Engr., Los Angeles, Calif.

[58a] Received by the Secretary, June 23, 1931.

[59] "Conditions de Stabilité des Barrages à Gravité en Béton," Extrait du *Bulletin* de la Société Belge des Ingénieurs et des Industriels.

[60] See, also, Professor Campus, "Note sur la Déformabilité des Barrages à Gravité" Extrait des *Bulletins* de l'Académie royale de Belgique (Classe des Sciences) Séance du 6 décembre 1930 (n. 12, pp. 1384–1395) (1931), and *Le Génie Civil*, January 3, 1931, p. 9.

The Usual Engineering Theory.—This assumes linear stresses, which satisfy the statical conditions, Equations (3) and (4). Since the bending formula is applied to horizontal sections in order to derive the magnitude of the linear stresses, it implies that these sections remain plane, when shear deformations are neglected. As already stated, this assumption is in error.

Theory of the Rigid Base.—This assumes that the foundation is unyielding in every direction. This is implied in the fact that Equations (25) and (26) were integrated from zero to H, which excludes deformations in the foundation, and, in Appendix III, the writer stated that no account had been taken of the irregularities introduced by the foundation. The late Professor Cain stated this explicitly in his discussion.

General Remarks.—The classical theory of elasticity is exact with the assumptions made. The ordinary engineering theory, which deals with a dam on a foundation, both of them elastic, is incorrect. As Dr. Vogt has shown, linear stresses curve the horizontal sections of the dam, concavity upward, while these same linear stresses curve the foundation in a complicated fashion, but in the main with the concavity downward. As stated by Dr. Vogt:

"The classical theory gives stresses which are greatly in disagreement with the compatibility equation at the foundation joint, and several large corrections are required."

Dr. Vogt has calculated approximately the additional stresses necessary to bring about similar deformations of the dam and its foundation, and checks the writer's results on the assumption that deformation due to water load only is involved. If the weight of the dam is included, he gets a smaller additional tension at the heel. He also calls attention to the possible effect of the dam being built by degrees and of the lower part partaking in the stresses due to weight along with the foundation. This brings up another complicated problem which may bear considerably upon the actual stresses in the foundation.

The late Professor Cain derives the conditions for minimum of internal work: $\frac{dL}{dC} = 0$, and $\frac{dL}{dF} = 0$, by means of Castigliano's theory, and states:

"These two equations show that the internal work is a minimum as stated by the author. This demonstration assumes the base of the dam unyielding."

He concludes:

"The English experiments first proved the ordinary 'inner-third rule' inadequate, and now Mr. Jakobsen confirms it by a close theoretical analysis."

In his discussion, Mr. Gore gives additional information regarding the English tests. Fig. 14 shows the piling up of stress at the heel of the dam, when the reservoir is empty. This was to be expected, but is probably somewhat too large in the model tests, due to the overhang when the reservoir is empty, shown in Fig. 13. In that respect, the writer's assumption of a rigid base is in error, because it gives nearly linear stress (see Article VII).

Mr. Gore, in collaboration with Mr. J. S. Wilson, conducted these tests. It was not the writer's intention to detract in any way from these very skil-

fully made tests, which to this day have not been surpassed or equalled. The writer's interest centered largely on the stresses at and near the foundation, where the English tests show tension at the heel. Mr. Gore has correctly understood that the writer assumed a rigid base and he concludes that this is the reason for the excessive tension found by the writer. Mr. Gore further concludes that the writer's assumption of a rigid base is not much nearer satisfaction than the straight-line law. After careful study of the discussions, the writer has concluded that the truth regarding tension at the heel is probably somewhere between the straight-line law and his own results.

There is, however, this important distinction, that the writer's error is on the side of safety, while the straight-line law unquestionably errs on the side of danger. This is a matter of considerable importance, because all are agreed on the uncertainties of the stress distribution at the base and yet this is the deciding factor in determining the width of the base and, therefore, the design of a gravity dam.

Mr. Gore's conclusion in favor of the linear stress distribution disagrees with his own and other tests, which led Professor Love to question the correctness of the classical theory.[61]

Mr. Mensch refers to Equation (14) for the internal work as a hypothetical equation, thus leaving it to be inferred that something is amiss with this equation. It is, however, the usual and accepted form for the work equation and is to be found in all standard works on elasticity (see, also, Professor Cain's discussion, in which this matter and the method of determining the least work, is carefully analyzed).

To replace the writer's Equation (15) Mr. Mensch offers a cubic parabola, but he makes no effort to show that this can be made to satisfy the equilibrium equations and the boundary conditions. His proposed curve (Fig. 16) shows a maximum at the down-stream face, no matter how wide the base, which is in contradiction to all tests of which the writer has any knowledge, and is otherwise improbable. Mr. Mensch takes the writer to task for finding tension in the up-stream face and for not mentioning that it cannot exist. The writer believes tension can exist and no doubt does exist in some dams, but he is agreed that the design should be made so that it will not occur. The writer agrees with Mr. Mensch regarding the danger of designing gravity dams with no allowance for uplift, such as was the case with the St. Francis Dam.

Mr. Jorgensen calls attention to the influence of water-soaking, the large variation in the modulus of elasticity, and to the influence of uplift, all of which are factors that make for uncertainties in any stress determination. They warn, or should warn, the designer that he must provide a liberal factor of safety to include a reasonable factor of ignorance.

Mr. Turner gives an interesting historical account of the development of the theory of bending. His statement that the writer has confused shear force with shear strain will be found incorrect upon further study. His assertion that the linear law at a section normal to the neutral axis follows from Hooke's law, is incorrect, as a reference to any text on elasticity will show.

[61] "The Mathematical Theory of Elasticity," Third Edition, p. 211.

An attempt to satisfy the equilibrium equations and the boundary conditions for an ordinary beam stressed in bending will also convince Mr. Turner of his error. Furthermore, Mr. Turner will find that the linear law applies to any section, horizontal or otherwise, of a triangular, homogeneous dam of indefinite length, and not only to the normal section. The writer noted this in his paper, "Stresses in Multiple Arch Dams",[62] and Professor Résal, in France, noted it in his paper[63] published in 1919. A few simple calculations will convince Mr. Turner of the correctness of this statement.

The tests by Mr. Barnes are of considerable interest. The writer has had an opportunity to check over the test data and his results are in substantial agreement with those of Mr. Barnes, except possibly at and near the down-stream toe where no deformation measurements are available. Mr. Barnes did not measure the horizontal deformations, but the assumption which he makes regarding n_x is no doubt quite close to the truth. It is also evident (see Equation (46)) that, for large values of n_y, an error in n_x is of little consequence.

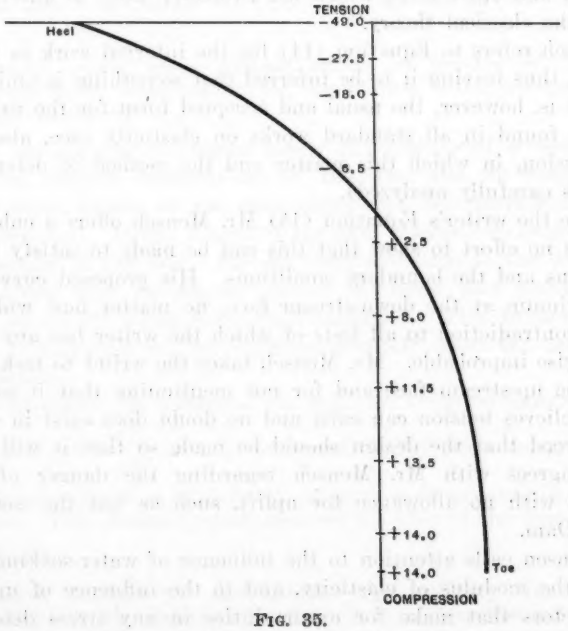

FIG. 35.

From Mr. Gore's data in Fig. 14, a stress curve for bending may be obtained, by subtracting the "reservoir-empty" stresses from the "reservoir-full" stresses. The result is shown in Fig. 35. The tension area checks the compression area within about 10 per cent. No doubt, the discrepancy is due to the small scale of Fig. 14 from which the stresses were measured, and

[62] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 294.

[63] "Formes et Dimensions des Grands Barrages en Maconnerie," *Annales* des Ponts et Chaussées, II, 1919.

does not reflect upon the accuracy of Mr. Gore's tests. The up-stream tension is 3.5 times the down-stream compression. The curve in Fig. 35 is of the same general shape as the writer's Fig. 6, in which the ratio between the maximum tension and the maximum compression is only 1.78. The curve in Fig. 35 is steeper in the up-stream parts of the dam and flatter in the down-stream portion than the writer's Fig. 6. The probable influence of the overhang (see Fig. 13), which is to increase the heel stresses for "reservoir empty," has already been mentioned. Both tests are in accord as to high tension at the heel near the base. Some of this tension will be compensated by high compression at the heel for "reservoir empty." On the other hand, there may be some doubt as to whether the English tests took sufficient account of the stresses due to the water load on the bottom of the reservoir (see Dr. Vogt's discussion and Fig. 25). Fig. 1, Plate 4, of the English tests[64] indicate that the reservoir load extended up stream about one-half the height of the dam. This, no doubt, is insufficient to produce even approximately full deformation caused by the reservoir load, and this loading certainly tends to produce tension at the heel.

In this connection the tests on models of arch dams to which Dr. Vogt referred, have now been made available.[65] In connection with tests on a model of the Gibson Dam, it is concluded:

"In the analysis of the arch elements the formulas for thick arches based on the assumption that plane sections remain plane, check better than the formulas based on a linear distribution of stresses. * * * In the analysis of the cantilevers the assumption of linear distribution of stresses over the cross-section seems to lead to discrepancies in the lower parts of the cantilevers, where the down-stream face has a slope of 0.8 to 1. The down-stream stresses are decidedly less than computed by such a linear distribution of stress, thus indicating a curved distribution of the stresses. The consequences of this, again, will be higher vertical tension at the up-stream face due to water load."

These tests agree with the English tests and those of Mr. Barnes. It is interesting to note that neither the arch stresses nor the cantilever stresses are linear, but are more nearly those found by assuming that plane sections remain plane.

Professor Hoadley calls attention to the uncomfortable feeling associated with work when the factor of safety approaches one, although little is at stake, and he approves of a more conservative viewpoint in gravity dam design. There is no doubt in the writer's mind that some gravity dams are in tension at the up-stream face and especially at the heel, namely, all those which have insufficient or no allowance for uplift. A number of such dams have already failed. The writer is well aware that the leaders in the profession never have countenanced such design methods, as the discussions published by the Society show.

Mr. Ely brings up a number of interesting points. Referring to Fig. 6, it seems that the fact that the section is not symmetrical would account for

[64] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II.

[65] Engineering Foundation, Committee on Arch Dam Investigation, Sub-Committee on Model Tests, J. L. Savage, M. Am. Soc. C. E., *Chairman*, June 1, 1931, p. 271, of multigraph copy.

the different stress distributions in the up-stream and down-stream halves of the dam. At the up-stream face the principal stress is vertical, while at the down-stream face it is parallel to that face.

Mr. Ely is no doubt correct in concluding that Equation (15) can be improved, possibly by introducing fractional exponents, as he suggests; perhaps also by writing Equation (15) so that the shape of the stress curves may vary with y. No doubt the model test of a dam 400 ft. high, which he contemplates, will bring out much of interest and will clear up obscure points. This investigation of the dam with inclined up-stream face is interesting and instructive.

Mr. Ely discussed uplift, and so did Professor Cain. The writer frankly admits that he is somewhat at sea as to how uplift stresses should be treated and just what effect they will have upon the stress distribution when considered in the light of the theory of least work. Recent discussions indicate that this is a complicated question, not to be answered in an off-hand manner.[66] However, dams designed on the straight-line theory should have a liberal uplift allowance; of that the writer feels certain.

The writer agrees with Mr. Gerry that the maximum compressive stress is no criterion of safety for ordinary gravity dams up to a height of 500 ft., or more, but the question of whether or not tension exists, or may exist, under the worst conditions to which the dam is exposed, is of great importance and is a question of stress. Mr. Gerry does well, however, in insisting that gravity dams must have a reasonable factor of safety against sliding.

The writer has already largely availed himself of the information supplied by Dr. Vogt, in which he explains so lucidly just why and to what extent the linear law fails at the foundation. He also shows that an equation of the form given by Equation (74), which is more general and includes the writer's assumption for n_y (Equation (15)), cannot satisfy all conditions. He also points out that convex curving of the down-stream face may produce tension; such curvature evidently should be avoided.

As a simple calculation will show Mr. Kalman's Equation (85) cannot possibly give values for the internal work as small as those found from the usual linear law. For that reason it is disqualified, *a priori*, and it is useless to determine the constants, a and b. Obviously, the writer's Equation (15) can equal Equation (10), the linear law, and, in Article VI, it was shown that the internal work found was less than that obtained from Equation (10). Mr. Kalman's repeated references to the true system leads the writer to remark that the Ritz method is of use only when the true system is not known. Fig. 31 is inapplicable, because the writer assumed a rigid base and it is immaterial how that is secured.

Professor Campus has apparently overlooked the basic assumption made by the writer, namely, that the base is unyielding while, as he has himself shown, the linear theory implies a curving of the base. This is the reason why Equation (45) leads to Equation (41) which is the linear solution,

[66] See articles in *Le Génie Civil*, by E. Batiele, March 24, 1928; by G. Pigeaud, March 31, 1928; by A. and Jacques Mesnager, April 7, 1928; and by G. Pigeaud, April 21, 1928. See, also, discussion by A. Floris, in *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 200.

while Equation (14) does not. In other words, when no assumption is made regarding the deformations of horizontal sections of an indefinitely high dam, the principle of least work must, and does, lead to the linear law.

Mr. Floris states that the writer has made exactly the same assumptions as were made in the classical theory of elasticity. He also has overlooked the fact that the writer assumed the base as unyielding, while the classical theory implies curvature, as shown by Dr. Vogt in Fig. 24.

Mr. Glover determines, by variational calculus, Equation (148), which is the same as the writer's Equation (42), and shows that if this equation is satisfied the internal work is a minimum. He has correctly understood that the writer assumed an unyielding base. The complete solution, of course, would have to include foundation deformations as well.

Referring to Mr. Werner's discussion, the boundary condition given by Equation (156) does not apply to the linear law, Equation (13), as both Professor Campus and Dr. Vogt have shown. It does apply to Equation (27), but since this does not satisfy Equation (150), which is the same as the writer's Equation (42), given and discussed in Appendix II, Equations (154) and (155) cannot be applied to Equation (27).

In practical application of the theory of elasticity, it is in general not possible to find an exact solution, which satisfies all the conditions of equilibrium and the geometrical continuity. One may then either determine the stresses from the equilibrium equations, in which case the geometrical continuity is not respected, and Equations (154) and (155) do not hold. If they are applied they must lead to contradictions, unless one accidentally hits upon the correct solution, or, one may find the stresses from Equations (154) and (155) and the geometrical boundary conditions, in which case the equilibrium equations are not likely to be exactly satisfied and cannot be applied.[67]

The writer has made an effort to reply to the various objections raised, but it is, of course, impossible adequately to cover the field gone over by so many able men. The discussions show a general agreement that the linear law errs on the side of danger and that there is more tension or less compression in the dam at the up-stream edge of the foundation than this law indicates. On the other hand, the writer's assumption and solution probably amounts to an over-correction, which errs on the side of safety. This is after all not such a bad feature, where so many uncertainties regarding uplift, internal tension due to shrinkage, etc., are involved, when the lives and property of so many people are at stake, and in view of the number of failures of gravity dams, which have occurred in the last decade. The assumption of a rigid base may lead to more nearly correct results by amending Equation (15) so that it allows for a change in shape of the stress curves with y, or it may be possible to include the foundation deformations in Equation (14) for the internal work. In any case, there is great need of carefully made tests which should include the water load on the reservoir and the consequent deformations at the heel.

[67] See Föppl's "Drang und Zwang," Vol. I, Second Edition (1924), p. 128.

In conclusion, the writer wishes to thank the many contributors for their assistance and interest and hopes that this paper will prove a contribution toward safer design, in order that the gravity dam designed so that it has insufficient or no uplift allowance, will be outlawed as it should be.

Due to an error in Equation (34), pointed out by Mr. Skytte, Equation (36) does not check Equation (27), as it should. When this correction is made, Equation (34) becomes:

$$n_y = -1.0579y + 1.8251x - 0.6558\,\frac{x^2}{y} + 0.05752\,\frac{x^3}{y^2}$$

$$t = +0.0146x - 0.2186\,\frac{x^3}{y^2} + 0.2876\,\frac{x^4}{y^3}$$

and when n_y and n'_y from Equation (35) are added, Equation (27) results, as it should. Fig. 5 is also changed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

A DISCHARGE DIAGRAM FOR UNIFORM FLOW IN OPEN CHANNELS

Discussion

By A. J. MATHESON, ESQ.

A. J. MATHESON,[10] ESQ. (by letter).[10a]—The writer has found this paper and the discharge diagrams presented therein, very interesting, particularly so because, since about 1916, he has been using a method of back-water computations which he developed, based on the same theory, but expressed by a different fundamental, or base-discharge curve, and extended upon somewhat different lines. Computations by either system, if based on the same precise data, should give exactly the same result.

In preparing his base-discharge curve for the example illustrated, Mr. Blanchard arranged the observations in groups (Table 2), in each of which the mean stage was approximately the same. He then plotted the mean elevation of each of these groups against the mean of the corresponding computed falls for a discharge of 8 000 cu. ft. per sec. The writer would suggest plotting the result of each individual observation, in obtaining the curve, because, by this process, any computed data which are obviously inconsistent can be eliminated and the general trend of the relations can be more closely analyzed. The conditions existing in the example he used for illustration are decidedly sensitive, since most of the recorded falls are well under 1 ft.

In the application of this method to any particular case, the field data required consist of a number of discharge measurements covering the range of flow, together with the corresponding records of the elevations of the water surface at the upper and lower ends of the reach. Since the accuracy of the final computations and curves is dependent upon the precision of these data, it would seem advisable to stress the following points:

(1) There must not be any variation in the volume of water flowing through the channel from the upper to the lower gauge, due to streams flowing into it or channels leading out.

NOTE.—The paper by Murray Blanchard M. Am. Soc. C. E., was published in January, 1931, *Proceedings*. Discussion on the paper has appeared in *Proceedings* as follows: May, 1931, by Messrs. Lynn Perry, J. B. McPhail, G. B. Pillsbury, Lynn Crandall, and Benjamin E. Jones.

[10] With Dominion Water Power and Hydrometric Bureau, Dept. of the Interior, Ottawa, Ont., Canada.

[10a] Received by the Secretary, May 28, 1931.

(2) The gauges must be tied in to a common datum so that the precise fall in the water surface between the two gauges may be obtained.

(3) The gauges, if possible, should be located so that during the time in which discharge measurements are being taken, there will be a fall of at least 2 ft. in the water surface between them. As the computed discharges are dependent on the square root of the fall, an error of, say, 0.10 ft. will cause a greater discrepancy with a total fall of 0.5 ft. than it would if the fall were 2 ft., or more.

(4) The gauges should be located so as to be as free as possible from the effect of local disturbances, such as eddies, wind, etc.

(5) The actual distance between the gauges is not important, as long as the flow is uniform throughout the channel.

(6) It is not essential that the cross-section of the channel should be the same throughout its length. It is essential, however, that there shall be no change in the channel cross-sections, such as erosion or silting of the sides or bottom, during or after the observations.

(7) Care should be taken to see that, during the time that measurements are being taken, the flow in the channel is not fluctuating and that the back-water conditions are not changing.

(8) This method of computation is not applicable to free falls, but its use is not confined to a straight-line, smooth water surface, between the recording gauges. The writer, using his own diagrams, computed the elevation at seven intermediate gauges along the rapids of the St. Lawrence River near Morrisburg, Ont., Canada, when levels had been taken after an ice jam had raised the water 10¾ ft. at the lowest gauge and 4 ft. at the upper gauge. His computed elevations at these seven gauges varied from an exact check, to a maximum difference of 0.14 ft. from the actual levels recorded by two independent parties of Government engineers. For the same flow under summer conditions the fall through these rapids is 10 ft. in a distance of about 3 miles.

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DISCUSSIONS

SPECIFICATIONS FOR BOUNDARY SURVEYS

PREPARED BY COMMITTEE OF SURVEYING AND MAPPING DIVISION ON BOUNDARY SURVEYS

Discussion

BY MESSRS. GEORGE M. BLEEKMAN, G. D. WHITMORE, AND WILLIAM W. MICHAEL

GEORGE M. BLEEKMAN,[2] ASSOC. M. AM. SOC. C. E. (by letter).[2a]—This work on boundary surveys speaks for itself as to quality. It was with great interest that the writer read the report and decided that engineers and surveyors have realized at last the necessity of a uniform code for their work. These specifications bring to light conditions generally existing and make valuable suggestions for performing real property surveys.

Due to the fine work which the Federal Government is doing in extending its triangulation system throughout the country, the engineer and the surveyor of to-day must gradually take advantage of the possibilities that this network affords. Monuments which have been recognized for years need not be abandoned; in fact, they can be more highly respected and located with reference to some system of control.

In a paper[3] entitled "Systematized Resurveys of Cities," Don W. Bingham, Assoc. M. Am. Soc. C. E., emphasizes the great need of the co-ordinate system. Many of the progressive cities are gradually obtaining this system, and the surveyor of the present and future will reap the many benefits. These systems can readily form the centers of different rectangular co-ordinate systems, and eventually they can be connected to the Geodetic System of the Government.

NOTE.—The Specifications for Boundary Surveys, Prepared by the Committee of Surveying and Mapping Division on Boundary Surveys, was published in May, 1931; *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Asst. Prof. of Geodesy and Surveying, Univ. of Michigan, Ann Arbor, Mich.

[2a] Received by the Secretary July 10, 1931.

[3] *Michigan Engineer*, March, 1931.

It is gratifying to note the recommendation of the Committee as to the accuracy which should be followed. The writer feels confident that very few local practitioners of the present day and age would fall below the ratio recommended. With the present equipment available and the type of men entering the field it will be highly respected.

The paragraph on writing descriptions was very much in order, and the writer hopes that something can be accomplished in the future whereby descriptions will be written only by those qualified for the work. Time after time the local practitioner is handicapped in his work by descriptions that are impractical or absolutely impossible to follow in the field. If the fee-holder, realtor, or lawyer could follow the man in the field and learn something of the operations necessary, he would soon realize the many difficulties that arise from faulty descriptions.

In preparing the Manual which is to follow, ample emphasis should be placed on the subject of monuments. In his paper, Mr. Bingham brings forth the slogan "let there be monuments," and the writer would like to add, "uniform monuments." The time has come when a uniform or standard monument should be set at all important points of a survey. Stakes, stones, plow points, etc., are monuments of the past, and the professional man of to-day should speak of one definite object when he says "monument."

G. D. WHITMORE,[4] ASSOC. M. AM. SOC. C. E. (by letter)[4a]—These specifications for boundary surveys establish an admirable foundation for the manual of practice which this Committee plans to produce. In considering them, one is impressed by the need of better National, State, and local organization among land surveyors, to the end that they as well as the public may be better informed as to the value of good surveys and methods of securing them.

Since the Committee invites discussion of its work at this stage, the writer offers some specific comment, preceded, however, by a general discussion as a background for the views expressed.

The subject of land surveying may be analyzed in either of two ways. Surveys may be classified according to their specific purposes, as control surveys, boundary surveys, topographic surveys, etc.; or they may be considered from the standpoint of the conditions, especially those of location, under which they are executed. Generally speaking, this latter classification applies to the subject of boundary surveys in that these may be separated into work in city or metropolitan areas and work done in all other localities, where property is originally not so valuable, and where boundary location may logically be less precise. Since the majority of the population in this country now lives under urban conditions, it seems proper to direct a large share of any consideration to boundary survey specifications to those applicable to city work. In the Manual of Technical Procedure for City Surveys, now in the final draft, by a Committee of the Surveying and Mapping Division,

[4] Vice-Pres. and Asst. Chf. Engr., R. H. Randall & Co., Inc., Toledo, Ohio.
[4a] Received by the Secretary August 5, 1931.

such specifications are detailed, and the writer believes it might be well to include some mention of them in the proposed Manual on Boundary Surveys.

The purpose of any boundary survey is: (1) To find or to prescribe the actual location upon the ground of one or more property boundary lines; (2) to mark the position of this line or lines by suitable monuments or visible references; and (3) to determine (*a*) the length and bearing of each line, and (*b*) the co-ordinates of each point, establishing mathematical relationships with other known points in the vicinity.

However, while the purpose of all boundary surveys is obviously to locate boundary lines, in city work boundary lines call for a high degree of accuracy, because of the small parcels of land involved, and the consequent greater precision required in the location of buildings thereupon. This relationship of property boundaries to building lines affords a reasonable basis for determining the requisite accuracy of boundary surveys in metropolitan areas, and the procedure necessary to secure such accuracy. Since it is not to be expected that any building or structure may be erected closer than within 0.02 ft. of the true position of the building or boundary line, the requirements of accuracy for boundary surveys in cities are satisfied, therefore, when any boundary line or point may be located or relocated within 0.02 ft. As a practical matter, this means that the required point should be located with this precision from one or more near-by points the physical and mathematical positions of which are known, because while it is theoretically possible, it is scarcely to be expected that the relocation of any lost point may be effected within 0.02 ft. of its original position by working in from control points several miles distant. To make relocation of this precision uniformly possible throughout a considerable area, it is necessary, therefore, as the Committee suggests, to establish a net of controlling triangulation, and to amplify this by precise traverses which distribute the benefits by supplying co-ordinates to numbers of intermediate points.

Because of the large proportion of boundary surveys that will necessarily be made in urban districts, it is the writer's thought that the Committee, in preparing its manual for practice, might well recommend a greater degree of accuracy of control than seems contemplated in the present draft of the specifications. In this draft, "an accuracy of 1 in 10 000 is recommended" for the actual surveys locating monuments and supplying descriptions of real property boundaries, and this precision is sufficient in most cases for individual property surveys. It is suggested, however, that the Committee include in its recommendations for control surveys, for establishing co-ordinates for and assisting the execution of such boundary surveys, smaller dimensions for limiting errors. The Manual of Technical Procedure for City Surveys, previously referred to, prescribes 1 in 600 000 as a limit of error in base-line measurement, and a base-to-base closure averaging 1 in 100 000. In actual practice this latter figure is commonly bettered. For instance, in the triangulation of Pittsburgh, Pa., errors developed by triangulation between eleven base lines average 1 in 200 000, and the base-to-base closure in Columbus, Ohio, is 1 in 700 000, etc.

In order to secure an accuracy of boundary-line location such that points may be relocated within 0.02 ft. from property points within the radius of approximately one block, it is essential to have triangulation of this greater accuracy, expanded by traverse with closing errors averaging 1 in 30 000. Traverses in the down-town section of Pittsburgh, established for controlling surveys of property boundaries as well as for all other survey purposes, have an average closure of 1 in 89 000 based upon triangulation of the accuracy cited and with triagulated points averaging one per square mile. Final adjustment of these traverses is effected by determining weighted co-ordinate values for "junction points," and then prorating the errors of the short lines, averaging 0.4 mile between these points. While the errors in latitude and departure thus distributed are very small, experience shows that after the most logical adjustment possible, additional lines, cutting up the areas into successive smaller units, may sometimes indicate errors in excess of the 0.02-ft. limitations.

The use of rectangular co-ordinates to define the exact location of property monuments is certainly to be recommended. A few cities, fortunate enough to have precise control surveys, have already made the use of rectangular co-ordinates mandatory by ordinance.

It is believed that the area over which a system of rectangular co-ordinates from one origin may be expanded, as recommended in the report, is rather small. It is entirely feasible, and in many cases desirable, to expand a rectangular co-ordinate system over an entire county, or group of counties, including an area of 1 200 to 1 400 sq. miles. At 20 miles from the origin the rectangular co-ordinates of a point are accurate with respect to the origin within 1 part in 90 000. The plane azimuths will vary from astronomic azimuths at a rate of about 40″ per mile, for distances east and west of the origin (the rate varies with latitude); but if an astronomic azimuth is desired it can be quickly derived from the plane azimuth furnished by the rectangular co-ordinates. The majority of recent city control surveys extend over an area much greater than the 40 or 50 sq. miles stipulated in the Committee's report and then make use of only one origin of plane co-ordinates. This plan appears to serve much more satisfactorily than would be the case were the large areas divided into units of 50 sq. miles, each with its own system of rectangular co-ordinates.

WILLIAM W. MICHAEL,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—The writer agrees with the Committee as to the importance of developing a campaign of education relative to accuracy of surveys, monuments, control, and records for boundary surveys.

Too often real property that was originally plentiful and that could be had for a nominal sum has developed into valuable and expensive land. The original and whatever subsequent surveys that were necessary, were often done in a haphazard manner; even monuments of a quasi-permanent nature were not established. Consequently, as this real property grew in value great diffi-

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[5a] Received by the Secretary August 17, 1931.

culties arose as to the location of the exact boundaries as established by the original survey. The writer cites these well-known facts to bring before the profession the importance of emphasizing to owners, attorneys, and other interested parties, the importance of permanent records, be they monuments, plats, or other information.

The writer has observed that even to-day all that is necessary to record a map in certain counties is to present it to the recorder or county clerk. No attention is given by these officials or their deputies as to its accuracy, the position of monuments, or, quite often, to the information it purports to indicate. Two ways may be open for solution to this problem, either education or legislation. If the Committee can bring before the public, in any manner, the facts relative to necessary requirements, they will have made a valuable contribution, not only to the profession, but to every owner of real property.

Certain States have legislation accurately defining the duties of the surveyor and map laws governing the requirements for record. It would seem to the writer that this is fundamental and most essential. Maps and descriptions offered for record should be scrutinized by competent engineers as to accuracy, monuments indicated, and clearness of description.

Contrary to professional ethics some surveyors follow the practice of retaining for their private information maps and plats made for transfer of real property. Such maps and plats should be required to be filed as permanent records accessible to any interested parties. The placing of permanent monuments at proper points should be emphasized and required. The owner of real property should be instructed and impressed with the value of correct surveys and descriptions. Too often they or future owners have discovered that the cheapest survey is the most expensive.

Co-operation between attorneys and engineers exists and should be furthered. The writer agrees with the Committee that it is the province and duty of the engineer and surveyor to prepare the description of real property. It is also the duty of the engineer or surveyor to familiarize himself with some of the legal aspects of description writing and deed interpretations.